The Ergun equation for pressure drop in a PBR

\[
\frac{dP}{dz} = - \frac{G}{\rho_g c D_p} \left( \frac{1 - \phi}{\phi^3} \right) \left[ \frac{150(1 - \phi)\mu}{D_p} + 1.75G \right]
\]

\( P = \text{pressure, lb}_f/ft^2 \ (\text{kPa}) \)

\( \phi = \text{porosity} = \frac{\text{volume of void}}{\text{total bed volume}} = \text{void fraction} \)

\( 1 - \phi = \frac{\text{volume of solid}}{\text{total bed volume}} \)

\( g_c = 32.174 \text{ lb}_m \cdot ft/s^2 \cdot lb_f \) (conversion factor)

\( = 4.17 \times 10^8 \text{ lb}_m \cdot ft/h^2 \cdot lb_f \)

(Recall that for the metric system \( g_c = 1.0 \))

\( D_p = \text{diameter of particle in the bed, ft (m)} \)

\( \mu = \text{viscosity of gas passing through the bed, lb}_m/ft \cdot h \ (\text{kg/m} \cdot \text{s}) \)

\( z = \text{length down the packed bed of pipe, ft (m)} \)

\( u = \text{superficial velocity = volumetric flow rate} \div \text{cross-sectional area of pipe, ft/h (m/s)} \)

\( \rho = \text{gas density, lb}_m/ft^3 \ (\text{kg/m}^3) \)

\( G = \rho u = \text{superficial mass velocity, lb}_m/ft^2 \cdot h \ (\text{kg/m}^2 \cdot \text{s}) \)

\[
\frac{dP}{dz} = - \frac{G(1 - \phi)}{\rho_0 g_c D_p \phi^3} \left[ \frac{150(1 - \phi)\mu}{D_p} + 1.75G \right] \left[ \frac{P_0}{P} \left( \frac{T}{T_0} \right) \frac{F_T}{F_{T0}} \right]
\]

\[
\frac{dP}{dz} = - \beta_0 \frac{P_0}{P} \left( \frac{T}{T_0} \right) \frac{F_T}{F_{T0}}
\]
Pressure drop along a packed bed reactor

\[
\frac{dP}{dW} = -\frac{\beta_0}{A_c(1-\phi)\rho_c} \frac{P_0}{P} \left(\frac{T}{T_0}\right) \frac{F_T}{F_{T0}}
\]

\[
\frac{dP}{dW} = -\frac{\alpha}{2} \frac{T}{T_0} \frac{P_0}{P/P_0} \left(\frac{F_T}{F_{T0}}\right)
\]

\[
\alpha = \frac{2\beta_0}{A_c\rho_c (1-\phi)P_0}
\]

The above always applies, especially for multiple reactions.

Now define \( y = \)

So for single reactions, where we have one conversion of interest, \( X \), then

\[
\frac{dy}{dW} = -\frac{\alpha}{2y} (1 + \varepsilon X) \frac{T}{T_0}
\]

So two differential equations in terms of \( X \) and \( P \).

Note for pipes without packing:

\[
y = (1 - \alpha_p V)^{1/2}
\]

\[
\alpha_p = \frac{4fG^2}{A_c\rho_0 P_0 D}
\]

\( f = \) Fanning friction factor and \( D = \) pipe diameter