The Ergun equation for pressure drop in a PBR

$$\frac{dP}{dz} = -\frac{G}{\rho g_c D_P}\left(\frac{1-\phi}{\phi^3}\right)\left[\overbrace{\frac{150(1-\phi)\mu}{D_P}}^{\text{Term 1}} + \overbrace{1.75G}^{\text{Term 2}}\right]$$

P = pressure, lb_f/ft^2 (kPa)

ϕ = porosity = $\dfrac{\text{volume of void}}{\text{total bed volume}}$ = void fraction

$1-\phi = \dfrac{\text{volume of solid}}{\text{total bed volume}}$

$g_c = 32.174\ lb_m \cdot ft/s^2 \cdot lb_f$ (conversion factor)

$= 4.17 \times 10^8\ lb_m \cdot ft/h^2 \cdot lb_f$

(Recall that for the metric system $g_c = 1.0$)

D_P = diameter of particle in the bed, ft (m)

μ = viscosity of gas passing through the bed, $lb_m/ft \cdot h$ ($kg/m \cdot s$)

z = length down the packed bed of pipe, ft (m)

u = superficial velocity = volumetric flow rate ÷ cross-sectional area of pipe, ft/h (m/s)

ρ = gas density, lb_m/ft^3 (kg/m^3)

$G = \rho u$ = superficial mass velocity, $lb_m/ft^2 \cdot h$ ($kg/m^2 \cdot s$)

$$\frac{dP}{dz} = -\underbrace{\frac{G(1-\phi)}{\rho_0 g_c D_P \phi^3}\left[\frac{150(1-\phi)\mu}{D_P} + 1.75G\right]}_{\beta_0}\frac{P_0}{P}\left(\frac{T}{T_0}\right)\frac{F_T}{F_{T0}}$$

$$\boxed{\frac{dP}{dz} = -\beta_0 \frac{P_0}{P}\left(\frac{T}{T_0}\right)\frac{F_T}{F_{T0}}}$$

Pressure drop along a packed bed reactor

$$\frac{dP}{dW} = -\frac{\beta_0}{A_c(1-\phi)\rho_c}\frac{P_0}{P}\left(\frac{T}{T_0}\right)\frac{F_T}{F_{T0}}$$

$$\frac{dP}{dW} = -\frac{\alpha}{2}\frac{T}{T_0}\frac{P_0}{P/P_0}\left(\frac{F_T}{F_{T0}}\right)$$

$$\boxed{\alpha = \frac{2\beta_0}{A_c\rho_c(1-\phi)P_0}}$$

The above always applies, especially for multiple reactions.

Now define *y =*

So for single reactions, where we have one conversion of interest, X, then

$$\boxed{\frac{dy}{dW} = -\frac{\alpha}{2y}(1+\varepsilon X)\frac{T}{T_0}}$$

So two differential equations in terms of X and P.

Note for pipes without packing:

$$y = (1-\alpha_p V)^{1/2} \qquad \alpha_p = \frac{4fG^2}{A_c\rho_0 P_0 D}$$

f = Fanning friction factor and D = pipe diameter