

# Process Control, 3P4

## Assignment 2

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**Assignment objectives:** gaining an excellent understanding of dynamic systems; using computer models and Laplace transforms to infer and describe the model behaviour over time.

### Question 1 [10]

For each transfer function below, what can you say about the time-domain function,  $H(t)$ , where  $0 \leq t \leq \infty$ ? Do not solve for  $H(t)$  analytically.

Your answers should include the initial value, final value, and a description of the stability, smoothness, and oscillatory behaviour.

1.  $H(s) = \frac{6(s+2)}{(s^2+9s+20)(s+4)}$

2.  $H(s) = \frac{10s^2-3}{(s^2-6s+10)(s+2)}$

3.  $H(s) = \frac{16s+5}{s^2+9}$

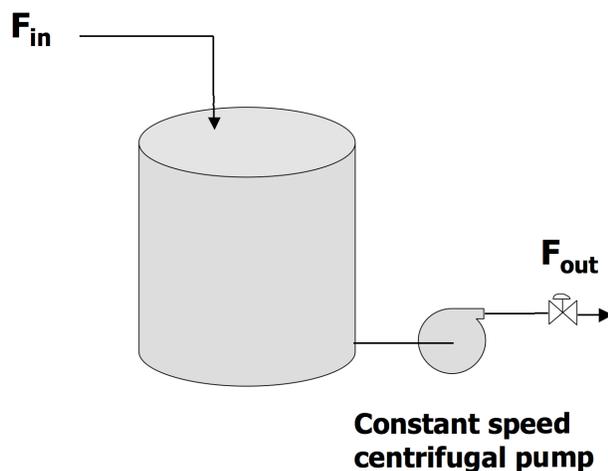
### Question 2 [4]

Find the inverse transform of:

$$Y(s) = \frac{s+2}{s(s+1)^2}$$

### Question 3 [20]

We want to determine the dynamic response of the liquid level in the open tank shown in the figure.



The flow entering is determined from an upstream process (we cannot control that flow rate). The flow out is determined by the valve percent opening, with a constant speed, centrifugal pump. (The level in the tank in this system does not significantly affect the flow out because of the high pressure supplied by the pump.)

Some data and details are given in the following:

- Initial flow in =  $2.0 \text{ m}^3 \cdot \text{min}^{-1}$
- Initial flow out =  $2.0 \text{ m}^3 \cdot \text{min}^{-1}$
- Total tank volume (when full) =  $10.0 \text{ m}^3$
- Initial tank level = 50% full
- Tank cross-sectional area =  $3.0 \text{ m}^2$
- Assume that the tank is initially at steady state with the flows in and out equal.

A step change is introduced in the inlet flow; it changes from  $2.0 \text{ m}^3 \cdot \text{min}^{-1}$  to  $1.0 \text{ m}^3 \cdot \text{min}^{-1}$ . No change is made to the valve opening.

1. Without using a computer or analytically derived expression, draw the plot of how you expect the liquid level to change once the step change is made. [There are no grades for this part of the question, so don't modify your graph later on].
2. Determine the actual dynamic response of the liquid level using a computer simulation.
3. Based on your graph and numerical results, what do you conclude about whether the level should be controlled using the feedback principle by adjusting the valve in the output after the pump?
4. The liquid is water at  $25^\circ\text{C}$ , and we want to measure the level within  $\pm 1$  percent of its true value. What level sensor would you recommend? Give a brief explanation with a citation for information used in your answer. [Hint: use information available at <http://pc-education.mcmaster.ca/> and follow links to the Instrumentation section].
5. The flow rate out of the tank must be measured with an accuracy of  $\pm 1$  percent from about  $0.25$  to  $2.0 \text{ m}^3 \cdot \text{min}^{-1}$ . What flow sensor would you recommend? Give a brief explanation with a citation for information used in your answer.

#### Question 4 [12]

A heat exchanger system has a differential equation below that shows how the temperature changes with a change in flow rate,  $q(t)$ .

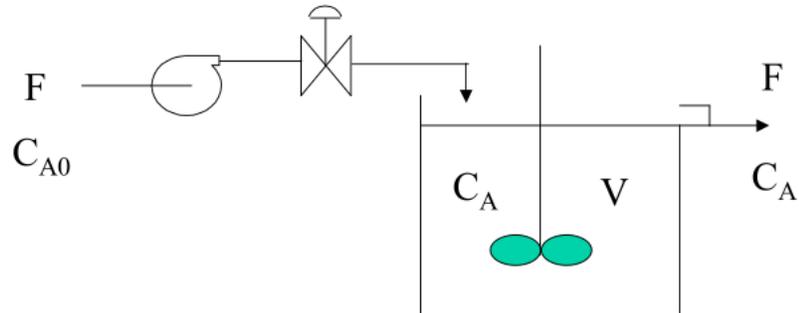
$$2 \frac{dT}{dt} = -T + 5q$$

The temperature,  $T$  and  $q$  are in deviation form already.

1.  $q(t)$  is changed from 0 to 2.0 at time  $t = 0$ . Sketch the response up to the point where the temperature reaches its new steady state.
2. What is this steady state?
3. How long does it take to reach within 0.1 degree of the new steady state.
4. If instead  $q(t)$  is changed from 0 to 4.0 at time  $t = 0$ . Sketch the response again, and report the new steady state value of the temperature.
5. Superimpose the plots from part 1 and part 4 of this question and explain why the curves have the same shape.

### Question 5 [15]

In this question, you will reinforce the material learned in Chemical Engineering 3E04. The system that you will consider is given in Figure 4. It is a CSTR where the reaction  $A \rightarrow B$  is occurring. You do not have to derive the component material balance, i.e., the differential equation in the figure.



$$r_A = \frac{-k_1 C_A}{(1 + k_2 C_A)}$$

$$V \frac{dC_A}{dt} = F(C_{A0} - C_A) + (V)(r_A)$$

$$V = 2.0 \text{ m}^3 \quad F = 1.0 \text{ m}^3 / \text{h} \quad k_1 = 1.0 \text{ h}^{-1} \quad k_2 = 1.0 \text{ m}^3 / \text{mole}$$

1. First, you must determine the *initial* value for the reactor concentration (which is the same as the concentration leaving the reactor),  $C_A$ . The system is initially at steady state, with  $C_{A,0} = 0.5 \text{ mol.m}^{-3}$ .

You may solve this part analytically or numerically. *Hint:* you will solve a quadratic equation.

2. Starting from this steady state, determine the dynamic response of the concentration  $C_A$  in the reactor if a step increase of  $C_{A,0}$  is made from  $C_{A,0} = 0.5 \text{ mol.m}^{-3}$  to  $C_{A,0} = 1.0 \text{ mol.m}^{-3}$ .

For example, the above occurs when your manager has asked you to increase the concentration of species B that you are producing.

3. How long does it take for  $C_A(t)$  to reach a new steady state?
4. You want to reach the new steady state faster. Explain how you can decrease the time taken. Perform a new simulation with the change(s) implemented, and superimpose the original time-domain plot from part 2 to show how much faster your response is.

*Work ahead:* in the next tutorial and assignment you will create deviation variables  $C'_A$  for outlet concentration, and  $C'_{A0}$  for inlet concentration. Create a linearized model, invert it with the Laplace transform, and compare the linearized (approximate) model to the actual model.

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END