# Process Control, 3P4

# Written midterm 1, 06 February 2014

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**Note:**

- You may bring in any printed materials to the midterm; any textbooks, any papers, *etc*.
- You may use any calculator during the midterm.
- **To help us with grading, please start each question on a new page, but use both sides of each page in your booklet**.
- You may answer the questions in any order on all pages of the answer booklet.
- This exam requires that you apply the material you have learned here in 3P to new, unfamiliar situations, which is the level of thinking we start to require from students that will be graduating or working in co-ops in a year from now.
- Any ambiguity or lack of clarity in a question may be resolved by making a suitable and justifiable assumption, and continuing to answer the question with that assumption(s).
- **Total marks**: 66 marks, 12.5% of course grade.
- Total time: 2 hours (nominally), though you have "infinite" time to complete it. There are 4 pages on the exam, please ensure your copy is complete.

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**Question 1 [11 = 1 + 5 + 2 + 3]**

1. The Laplace transform and its inverse are:
   (a) the same
   (b) unique functions
   (c) defined at values for $t < 0$
   (d) only defined for complex functions of $s$
2. Give the time-domain solution for $h(t) = \ldots$ for the following differential equation system

$$\tau \frac{dh}{dt} = Km(t) - h(t) \qquad \text{where } m(t) = \begin{cases} 0 & \text{for } t < 2 \\ 3 & \text{for } t \geq 2 \end{cases}$$

   You may assume that $h(t)$ and input $m(t)$ are deviation variables.
3. Give an example of a process with zero gain.
4. Sketch a rough time-domain plot of $y(t)$ for the following expression written in Laplace transform. The plot must include suitable numeric values on the axes.

$$y(s) = \frac{6e^{-4s}}{s\,(9s+1)}$$

*Solution*

1. The Laplace transform and its inverse are *unique functions*.

2. The insight into solving this problem comes from recognizing it is a standard first-order system, where the input is a step input of magnitude 3, occurring at time 2. Because of the discontinuity, you cannot integrate this analytically. You must use Laplace transforms, and the time-delay function.

$$\tau \frac{dh}{dt} = Km(t) - h(t) \qquad \text{where } m(t) = \begin{cases} 0 & \text{for } t < 2 \\ 3 & \text{for } t \geq 2 \end{cases}$$

$$\tau s H' s(s) + H'(t=0) = KM'(s) - H'(s) \qquad \text{where } M'(s) = \frac{3}{s} e^{-2s}$$

   Now $H'(t = 0) = 0$, because we are told they are deviation variables, so we have:

$$H'(s)\,(\tau s + 1) = \frac{3Ke^{-2s}}{s}$$

$$H'(s) = \frac{3Ke^{-2s}}{s\,(\tau s + 1)}$$

$$\mathcal{L}^{-1}\left[H'(s)\right] = \mathcal{L}^{-1}\left[\frac{3Ke^{-2s}}{s\,(\tau s + 1)}\right]$$

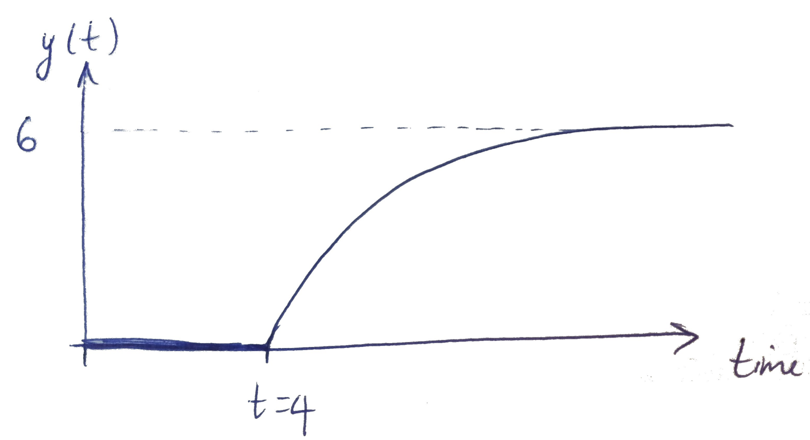
$$\text{Now } \mathcal{L}^{-1}\left[\frac{3K}{s\,(\tau s + 1)}\right] = 3K\left[1 - e^{-t/\tau}\right]$$

$$\text{So } \mathcal{L}^{-1}\left[\frac{3Ke^{-2s}}{s\,(\tau s + 1)}\right] = 3K\left[1 - e^{-(t-2)/\tau}\right].\mathcal{S}(t-2)$$

   using the rules we established in class for time delay inversion, and $\mathcal{S}(t - 2)$ is a unit step that occurs at time $t = 2$.

3. Any process where the output is unrelated to the input. Several interesting examples were provided in the midterm.

4. Similar to the question in part 2, we see this will be a regular, first-order response to a step input, but with a delay of 4 units. From the final value theorem we can tell the final value will be 6.



**Question 2 [11 = 3 + 5 + 2 + 1]**

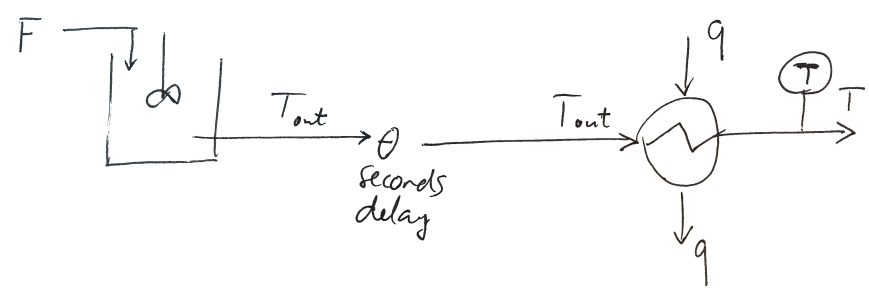
An exothermic stirred tank reactor has an inlet feed, given by flow $F$. The outlet temperature, $T_{\text{out}}$ shows a first order dynamic response when increasing this flow.

Downstream from the reactor is a heat exchanger, but the pipe connection from the stirred tank to the heat exchanger has approximately $\theta$ seconds of delay. The heat exchanger operates by using chilled water, which enters at a constant flow rate of $q$. The hot stream from the reactor enters the heat exchanger and then this exits with temperature, $T$. The dynamics relating the hot stream inlet temperature to the outlet temperature, $T$, are also first order.

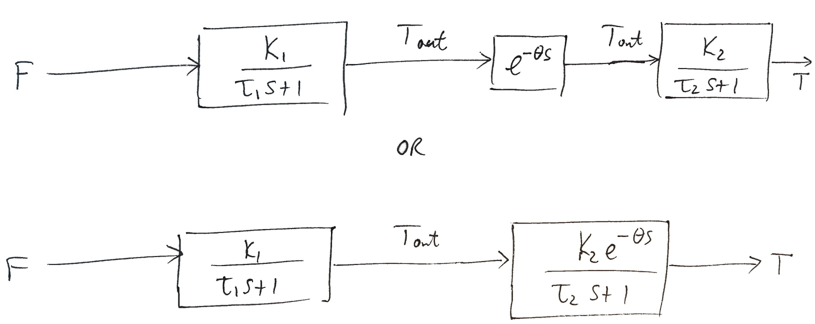
1. Draw a rough engineering diagram of the system (e.g. a piping an instrumentation diagram) with all the relevant symbols.
2. Draw a block diagram of the system, using one block per unit operation, starting from the feed flow rate $F$ as the input, up to the exiting temperature, $T$. In your block diagram enter all the transfer function and dynamic information you know, using appropriate symbolic representation learned in the course so far.
3. Write a single symbolic transfer function relating the incoming flow, $F$ to the exiting temperature, $T$.
4. What will be the order of the transfer function in part 3?

*Solution*

1. It is an important skill to be able to translate your process to a drawing




2. And a further skill to convert the drawing to a sequence of transfer functions




The time delay may appear either as a separate block, or merged with the second block.

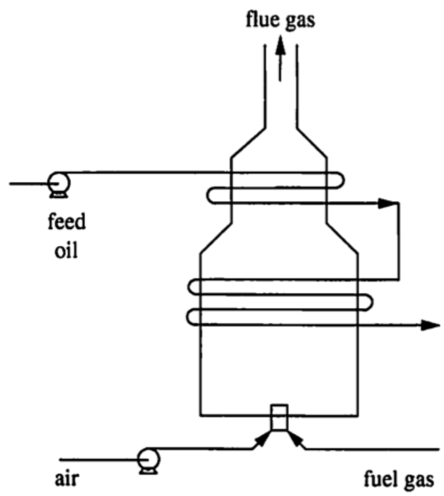
3. The combination of the two transfer functions is the product of the individual systems:

$$\frac{T(s)}{F(s)} = \frac{K_1 K_2 e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

4. It will be second order overall, and likely will be overdamped (most real engineering systems are; *why*?)

**Question 3 [8 = 4 + 4]**

Below is a diagram of fired heater used to pre-heat an oil stream before it is used in a different part of the process.




1. State 2 control objectives that would be considered when designing and forming the control loops for this process.
2. For one of your chosen objectives, very clearly specify what the manipulated variable and what the controlled variable would be, and justify (explain) your choice clearly.

*Solution*

1. There are several objectives that are relevant for control on this process. For example:
   - Safety, e.g. ensuring there is sufficient oxygen in the furnace, in excess of that required, to ensure complete combustion and avoiding CO production
   - Equipment protection: e.g. (a) ensuring the cold feed oil supply is sufficiently fast enough, to prevent flames from melting the tubular pipes in the furnace. Another example (b) would be to ensure the opening of the flue gas is enough to prevent excessive pressure build-up inside the furnace.
   - Economic operation: ensuring the right amount of fuel is supplied, and not too much, to achieve the desired exiting set point temperature.
   - Similar ideas can be developed for the other objectives of control systems.

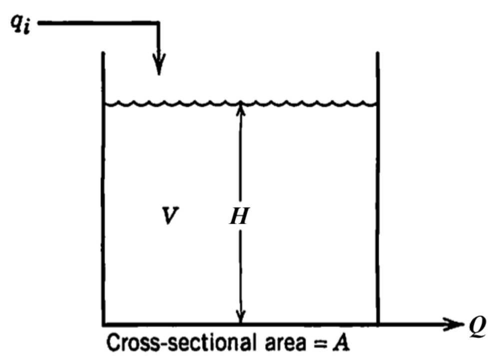
2. For example:

 - *Safety*: MV = air flow and CV = oxygen percentage in the furnace (to make sure it is at, or exceeds, a certain minimum)
 - *Equipment protection*:
   (a) MV = feed oil flow rate valve position; CV = feed oil flow rate.
   (b) MV = percent opening of damper in the flue gas, while CV = pressure inside the fired heater.
 - *Economic operation*: MV = fuel amount; CV = exit temperature of oil

**Question 4 [14 = 5 + 2 + 7]**

The Laplace transfer function for the tank height, $H$, when the outlet flow, $Q$, is varied is given by:

$$\frac{H'(s)}{Q'(s)} = -\frac{100}{20s+1}$$




The height is measured in inches, and the outlet flow, $Q$ is measured in liters per minute, and the inlet flow is $q_i$, a constant value. The variables $Q'$ and $H'$ are deviation variables.

1. What is the gain of the process, including units? In your answer, explain why the gain's value makes sense.
2. What is the time constant of the process, including units?
3. A step change, of unit magnitude is made to the outlet flow. What **change** in height will be observed in the tank after:
   (a) 0 minutes
   (b) 20 minutes
   (c) 40 minutes
   (d) 60 minutes

(e) 80 minutes

(f) 100 minutes?

Draw a plot of the tank's height response, $H'(t)$, based on your numeric values above. Mark where the 6 points are on your plot.

*Solution*

1. The gain of the process is $-100\,\dfrac{\text{in}}{\text{L/min}} = -100\,\text{in.min.L}^{-1}$.

   The value and the sign makes sense, because as the outlet flow increases, the tank's height will drop. (Note, variables are always in deviation form).

2. The time constant in 20 minutes.

3. For a unit step increase in flow, i.e. increase the flow by an additional $1\,\text{L.min}^{-1}$, then we can write, $Q'(s) = \dfrac{1}{s}$.

   This means the height will respond as $H'(s) = -\dfrac{100}{20s+1} \cdot \dfrac{1}{s} = \dfrac{-100}{s(20s+1)}$, or in the time-domain, $h'(t) = -100\left[1 - e^{-t/20}\right]$ inches.

   (a) When $t = 0$, we have $h'(0) = 0$ inches.

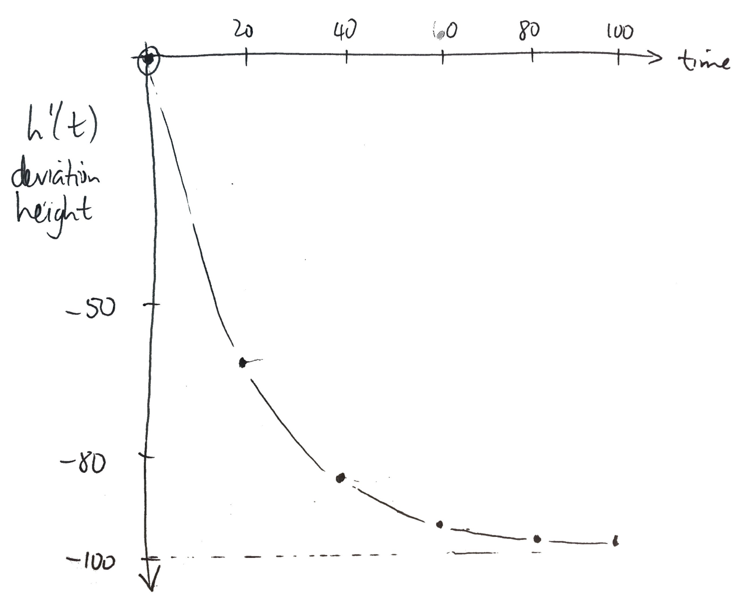
   (b) When $t = 20$, we have $h'(20) = -63.2$ inches

   (c) When $t = 40$, we have $h'(40) = -86.5$ inches

   (d) When $t = 60$, we have $h'(60) = -95.0$ inches

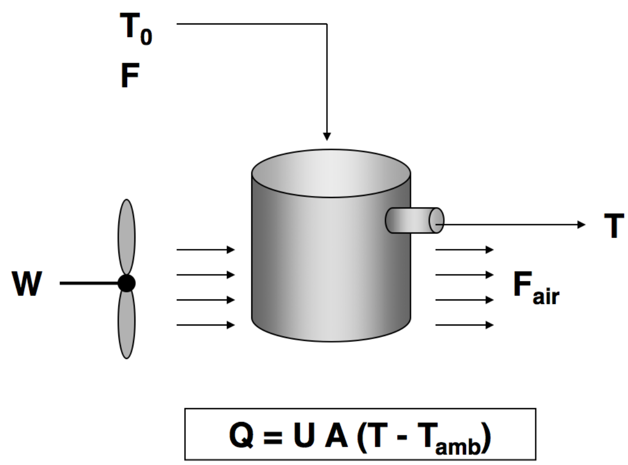
   (e) When $t = 80$, we have $h'(80) = -98.2$ inches

   (f) When $t = 100$, we have $h'(100) = -99.3$ inches



**Question 5 [19 = 5 + 1 + 10 + 3]**

A metallic stirred tank has a hot stream flowing into it, at temperature $T_0$; the stream leaving has a temperature, $T$. We want to judge how effective it will be to cool the tank when using a fan; since the inlet temperature is always varying, and we have to control the outlet temperature.




1. Show the ODE describing the temperature leaving the tank is:

$$V\rho C_p \frac{dT}{dt} = F\rho C_p(T_0 - T) - UA(T - T_{\text{amb}})$$

   where the heat capacity is $C_p$, the liquid density in the tank is $\rho$, the constant volume in the tank is $V$, the area available for heat transfer is $A$, the heat transfer coefficient is $U$, and the (assumed constant) ambient temperature is $T_{\text{amb}}$. The inlet flow, $F$ is also assumed constant.

   *In your derivation, be clear on which assumptions you are making.*

2. What are the units for each term in the ODE?

3. The motor powering the fan has a DC coil, so we can adjust the voltage, which will affect the amount of heat transfer. It has been found to be of the form $U = aW^{0.5}$, where $W$ is the voltage, and $a$ is a constant.

   As a step towards our goal, show how you would linearize the term: $UA(T - T_{\text{amb}})$.

4. We want to design a control system that will solve the goals of this problem.

   (a) What will be the manipulated variable?

   (b) What will be the controlled variable?

   (c) What will be considered the disturbance?

*Solution*

1. This question requires you to use the most basic idea in chemical engineering: the energy/enthalpy balance.

   Accumulation = in − out

Starting from this point:

$$\frac{d\left(V\rho C_p(T-T_{\text{ref}})\right)}{dt} = F\rho C_p(T_0 - T_{\text{ref}}) - F\rho C_p(T - T_{\text{ref}}) - UA(T - T_{\text{amb}})$$

Then making several simplifying assumptions:

- the tank is isothermal, and well mixed so the wall temperature is $T$ (reasonable for a CSTR)
- assume constant $\rho$ and $C_p$ (both are reasonable)
- constant $V$ and $F$ and $T_{\text{amb}}$ (all three are given as constant)
- assume constant $A$ (reasonable, the tank's external area doesn't change)

This leads to:

$$V\rho C_p\frac{dT}{dt} = F\rho C_p(T_0 - T) - UA(T - T_{\text{amb}})$$

Any term that is not constant, is time-varying, by definition. In this case, that means $T$, $T_0$ and $U$ would vary with time [part 3 of the question confirms that $U$ varies with time, because the question states that the voltage varies].

2. Each term has units of Joules/Time = Watts. If you defined the units of each variable in the equation, that was also OK.

3. The term $UA(T - T_{\text{amb}}) = UAT - UAT_{\text{amb}}$. In the first part, both $U$ and $T$ are time-varying, so this is nonlinear, and in the second part, the $U$ portion is nonlinear.

   Use subscript $s$ to denote steady state values:

$$UAT = aW^{0.5}AT \approx aW_s^{0.5}AT_s + 0.5aW_s^{-0.5}AT_s(W - Ws) + aW_s^{0.5}A(T - T_s)$$

   and

$$UAT_{\text{amb}} = aW^{0.5}AT_{\text{amb}} \approx aW_s^{0.5}AT_{\text{amb}} + 0.5aW_s^{-0.5}AT_{\text{amb}}(W - Ws)$$

   so combine these two together:

$$\begin{aligned} UAT - UAT_{\text{amb}} &\approx aW_s^{0.5}A(T_s - T_{\text{amb}}) + 0.5aW_s^{-0.5}A(T_s - T_{\text{amb}})(W - Ws) + aW_s^{0.5}A(T - T_s) \\ &\approx aW_s^{0.5}A(T_s - T_{\text{amb}}) + 0.5aW_s^{-0.5}A(T_s - T_{\text{amb}})W' + aW_s^{0.5}AT' \end{aligned}$$

4. (a) MV = voltage, $W$

   (b) CV = temperature, $T$

   (c) DV = inlet temperature, $T_0$

**Question 6 [3]**

You are working at a waste-water treatment plant, and the operator is complaining that the amount of oxygen in the water (dissolved oxygen level) is too low, below the required amount. He said: “the feedback control system is broken; please fix it”.

What **list of specific instructions** will you give to your summer student to go investigate whether the control system actually is broken?

*Solution*

Any answer that includes 3 of the 4 essential elements of a feedback control system is considered full grade. Having multiple points related to only one element for the control system would not be suitable.

Remember, the question asks you to be specific: this is a summer student who has to implement your work. This is important for your professional practice in the future: you will always be working with non-chemical engineers.

You cannot say: “check the sensor”; rather say: “check that the sensor reading displayed in the control room matches the dissolved oxygen value measured with the portable DO meter”. That will ensure the sensor is working, as well as that the sensor value is correctly displayed and used by the feedback control system.

You cannot say: “check the valve”; there are many valves in a plant. Furthermore, what should they check about the valve? That it is there? That it is open or closed?

It is good practice to learn to be specific in your instructions, so that non-chemical engineers work well with you.

For example, you will have to ask the student to inspect:

1. The sensor (the purity analyzer) to ensure that it is working as expected [e.g. use a sample with a known/reference DO and measure it with the existing sensor]
2. The controller itself (either a computer, or some digital electronics that implements the feedback control algorithm) should be sending a signal to the valve. Most modern control systems have a digital display of the output value being sent to the valve. For example, ask the student to make a set point change up, then monitor that the valve is being asked to open more than its current value.
3. The valve that adjusts the manipulated variable should be working correctly. Is it stuck shut? Is the signal being received by the valve. The student will have to go to the valve and check that oxygen is flowing through it.
4. Is the set point specification is correctly entered in the control system? This can be typically seen in the operating room.

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**The end.**