

$$V \frac{dC_A}{dt} = F(C_{A0} - C_A) - V k C_A^2$$

$$V \frac{dC_A}{dt} = F(C_{A0} - C_A) - [V k C_{As}^2 + 2V k C_{As}(C_A - C_{As})]$$

$$\frac{dC'_A}{dt} + \frac{1}{\tau} C'_A = \frac{K_p}{\tau} C'_{A0}$$

$$\tau = \frac{V}{F + 2V k C_{As}} \quad K_p = \frac{F}{F + 2V k C_{As}}$$

$$sC'_A(s) - C'_A(t)|_{t=0} + \frac{C'_A(s)}{\tau} = \frac{K_p}{\tau} C'_{A0}(s)$$

$$\text{For } C'_A(t)|_{t=0} = 0$$

$$C'_{A0}(s) = \frac{\Delta C'_{A0}}{s}$$

$$C'_A(s) = \frac{K_p \Delta C_{A0}}{s(\tau s + 1)}$$

Table 4.1, entry 5

$$C'_A(t) = K_p \Delta C_{A0} (1 - e^{-t/\tau})$$

Formulate Model Based on Conservation Balances and Constitutive Relationships

- “Exact” dynamic behavior described by model

Linearize Nonlinear Terms

- Easier to solve analytically
- Useful for determining some properties, e.g., stability

Numerical Simulation

- Determine the complete transient response

```
% Pseudocode for Euler's integration
T(1) = 0 % Initialize
CA(1) = CAINIT
FOR N = 2: NMAX
IF N > NSTEP, CA0 = STEP, END
DER = (F/V) * (CA0 - CA(N-1))
- K * (CA(N-1))^2
CA(N) = CA(N-1) + DELTAT * DER
T(N) = T(N-1) + DELTAT
END
```

Express in Deviation Variables

- Required so that transfer functions are linear operators

Take the Laplace Transform

$$\text{Transfer Function: } \frac{C_A(s)}{C_{A0}(s)} = \frac{K_p}{(\tau s + 1)} = G(s)$$

$$\text{Final Value: } \lim_{t \rightarrow \infty} C'_A(s) = \lim_{s \rightarrow 0} s C_A(s)$$

$$= \lim_{s \rightarrow 0} s \frac{\Delta C_{A0}}{s} \frac{K_p}{(\tau s + 1)}$$

$$= K_p \Delta C_{A0}$$

$$\text{Stability: Pole } s = \frac{-1}{\tau} < 0$$

∴ stable

Frequency Response:

$$AR = |G(j\omega)| = \frac{K_p}{\sqrt{1 + \omega^2 \tau^2}}$$

$$\Phi = \angle G(j\omega) = \tan^{-1}(-\omega\tau)$$

Solve Analytically

(Invert to time domain)

- Use Table 4.1
- Expand using partial fractions
- General initial conditions and input forcing

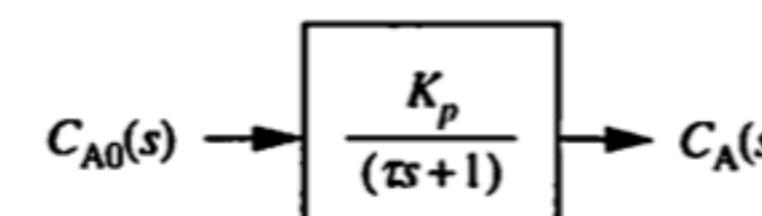
Results: Complete transient of the linearized system

Formulate Transfer Function

(Do not solve for entire dynamic response)

- Set all initial conditions to zero
- Draw block diagram of system
- Derive overall transfer function using block diagram algebra

Results: Final value, stability, and frequency response



Shows cause-effect direction

FIGURE 4.11

Steps in developing models for process control with sample results for a chemical reactor.