Process Control, 3P4

Assignment 1

Kevin Dunn, kevin.dunn@mcmaster.ca **Due date: 20 January 2014**

Assignment objectives: understanding about feedback systems; dynamic models

Question 1 [4]

Select a system from everyday life that employs feedback in its operation. Describe the system and how feedback is used. A sketch of the system is required.

Solution

Any system might be used that has feedback control. Here are some examples:

- a toaster that turns off automatically once a maximum temperature is reached.
- a rice cooker: when the temperature is that of boiling water, it continues to supply heat, until the temperature exceeds boiling, and the rice is considered to be cooked.
- a refrigerator, or home heating system: both use on-off control to maintain a temperature within a given lower and upper bound.
- cruise control in a car

Many other options can be considered, including those “feedback” control systems that are part of the human body.

Question 2 [8]

Select a process that you have studied in a previous course in chemical engineering (e.g. heat exchangers, separation process, reactors) and describe the objectives that should be associated with each of the seven categories of control objectives described in Chapter 2 of the textbook. A sketch of the process is required.

Solution

Note: The question was quite clear, and it was emphasized in tutorials as well: you have to select a process from another prior course. It is not correct to simply copy the example we covered in class and repeat it as your answer. This course, and all courses at McMaster, are about application of your knowledge. Reusing an example from class does not demonstrate that ability.

The seven categories of control are:

1. safety of the people and neighbours
2. protecting the environment
3. protection the equipment
4. smooth process operation
5. product quality
6. profitability
7. monitoring and diagnosis.

It is unlikely that one control loop will achieve all 7 objectives, which is why you were asked to select a process. One loop on that process will be for one/more objectives, while another loop will be for other objectives.

For example, in a furnace we will have a loop to ensure the level of the hydrocarbon being heated is always above a minimum value, so we do not melt the tubes with the flames. Empty tubes will melt, since the tubes are made with thin metal to maximize heat transfer. This loop addresses safety, and equipment protection.

A safety release valve (a type of valve that will automatically open when a minimum pressure is exceeded), is there to protect the equipment, and also divert the hydrocarbon to a safe location, so it is not released into the environment.

Temperature control loops in the furnace ensure that enough heat is supplied to obtain a stable, and almost constant outlet temperature. If this loop is well controlled, then the stable target temperature means we do not need overheat the hydrocarbon, and we operate more profitable (use only just the required amount of fuel).

Finally the furnace may have several temperature sensors and fuel flow sensors, so we can monitor fouling of the tubes over a long period of time. Once we notice we have to use an unacceptably large amount of fuel to achieve the same final temperature, we know the tubes are fouled. We can stop the furnace, and clean the pipes from accumulated fouling.

Question 3 [4]

What is the economic cost of an error of 0.5% in the flow rate for a pipeline that carries 100,000 barrels/day of crude oil? (assume the error is not in your favour, i.e. it really is a cost, and not a profit)

Solution

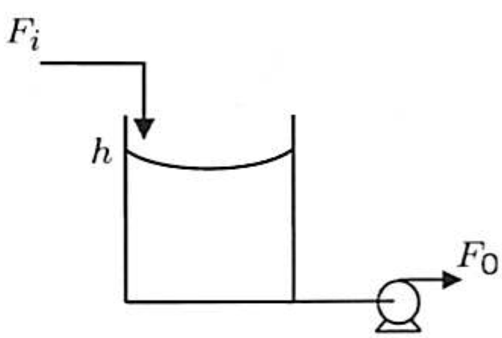
The cost of 0.5% error implies our flow sensor is reading 0.5% less than it should if we are selling the product; or it is reading 0.5% more than it should, if we are buying the crude oil.

Either way, at the current crude oil price of around \$110/barrels, this converts to a cost of $0.005 \times 100,000 \times 110 \approx$ $\$55,000$ per day. Any numbers around \$50,000 dollars is acceptable.

This indicates the strong financial incentive to have a flow sensor that is precise with much lower degree of error.

Question 4 [20]

Consider the following tank system:



1. Write out the mass balance for the tank in the form:

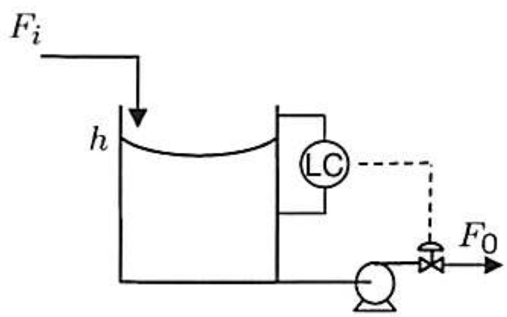
$$\text{rate of accumulation of mass} = \text{rate of mass flow in} - \text{rate of mass flow out}$$

   so that you obtain a differential equation relating height h, flows F_i and F_0, and the cross-sectional area, A. The flows are considered to be volumetric flows.

2. Let the flows be $F_i = 10\,\text{m}^3.\text{min}^{-1}$ and $F_0 = 11.5\,\text{m}^3.\text{min}^{-1}$, the area is $A = 2\,\text{m}^2$, and the current height in the tank is 3 m.

   Solve the differential equation by hand to determine an analytical expression of the tank height as a function of time, i.e. $h(t) = ...$

3. Sketch your function over time, for a duration of 3 minutes.
4. What is the tank level at $t = 2$ minutes?
5. Create a simulation of the system in MATLAB, and integrate the ODE using the MATLAB solver `ode45(...)`, which you learned about in Chemical Engineering `3E04`. Print and attach your plot, showing that your simulation matches the solution to part 3 and 4 above.
6. Now we add a simple controller to the system, a *proportional controller*.



A proportional controller modifies the valve position, *in proportion to* the error, or deviation from set point. The set point is $h_s = 2.0\,\text{m}$. When there is no deviation, the controller makes no change to the valve. If there is a high deviation from set point, it makes a greater change. The control algorithm can be represented as:

$$F_0(t) = F_{0,s} + K_c(h - h_s)$$

where the error term is $h - h_s$, and the controller tuning constant is $K_c = 8\,\text{m}^2.\text{min}^{-1}$. The term $F_{0,s} = 10\,\text{m}^3.\text{min}^{-1}$ represents the amount of flow leaving the tank when there is no error (i.e. when the height is equal to the set point).

Add this equation to your simulation from part 5 (either add it as a new equation - *preferred* - or substitute it into the previous equation).

Now re-simulate the height in the tank over time for a period of 3 minutes. What do you notice about your simulation, compared to part 5?

Print out and attach your simulation.

Solution

1. The ODE derivation is given below. If we assume constant density, then we get cancellation of ρ:

$$\frac{d\,(\rho A h)}{dt} = \rho F_i - \rho F_0$$
$$\frac{dh}{dt} = \frac{1}{A}\,(F_i - F_0)$$

2. Integration by hand, analytically:

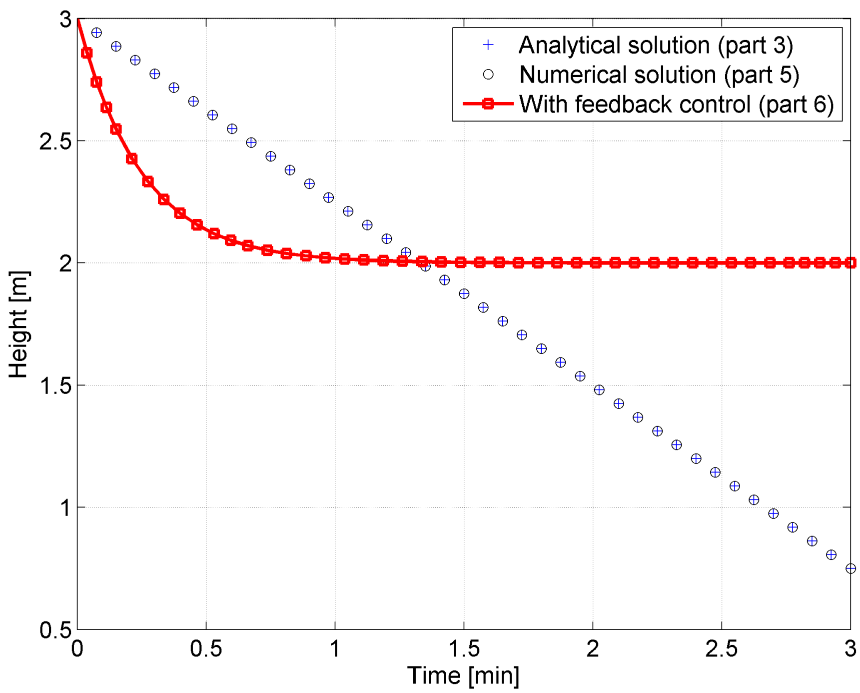
$$\frac{dh}{dt} = \frac{1}{A}\,(F_i - F_0)$$
$$\int_{h_0}^{h(t)} dh = \int_0^t \frac{F_i - F_0}{A}\,dt$$
$$h(t) = h_0 + \frac{(F_i - F_0)}{A}t = 3.0 + \frac{(10 - 11.5)}{2}t$$
$$h(t) = 3.0 - 0.75t$$

where time, t, is in units of minutes.

3. A figure of the trajectories is shown below, after part 6.

4. The tank level is $3 - 0.75(2) = 1.5$ meters after 2 minutes.
5. A figure of the trajectories is shown below, after part 6.
6. The code and behaviour of the system is shown below. Source is also included (though not required for the assignment submissions).

   The main difference is that the system reaches the desired steady state height of 2.0m. In part 5 the system was unstable, heading towards running empty. The system is stabilized with the feedback controller given by $F_0(t) = F_{0,s} + K_c(h - h_s)$. This control system adjusts the inlet flow in proportion to the deviation from set point, i.e. in proportion to $h - h_s$.



ODE_driver_assign1.m

```
% Integrating ODEs
% ------------------

% The independent variable always requires an initial and final value:
indep_start = 0.0; % minutes
indep_final = 3.0;

% Set initial condition(s): for integrating variables (dependent variables)
h_depnt_zero = 3.0;     % i.e. h(t=0) = 3.0

IC = [h_depnt_zero];

% Integrate the ODE(s):
[indep1, depnt1] = ode45(@assign1_Q4_part5, [indep_start, indep_final], IC);
[indep2, depnt2] = ode45(@assign1_Q4_part6, [indep_start, indep_final], IC);

% Plot the results:
clf
plot(indep1, 3 - 0.75 .* indep1, 'b+')
grid('on')
hold('on')
```

```
plot(indep1, depnt1(:,1), 'ko')
plot(indep2, depnt2(:,1), 'rs-', 'linewidth', 2)
ax = gca;
set(ax, 'FontSize', 14)

xlabel('Time [min]')
ylabel('Height [m]')
legend('Analytical solution (part 3)', ...
       'Numerical solution (part 5)', ...
       'With feedback control (part 6)')
print('-dpng', '3P4-assign1-plots.png', '-r300')
```

assign1_Q4_part5.m

```
function d_depnt__d_indep = assign1_Q4_part5(indep, depnt)

% Dynamic balance for the CSTR height

%    indep: the independent ODE variable, such as time or length or the reactor
%    depnt: a VECTOR of dependent variables
%
%    Returns:
%
%    d(depnt)
%   ---------- = a vector of ODEs
%    d(indep)

% Assign some variables for convenience of notation: one row per DEPENDENT variable
h = depnt(1);

% Constant and other equations
A = 2.0;    % cross-sectional area [m^2]
Fi = 10.0; % inlet flow rate [m^3/min]
F0 = 11.5; % outlet flow rate [m^3/min]


% Output from this ODE function must be a COLUMN vector, with n rows
% n = how many ODEs in this system?
n = numel(depnt); d_depnt__d_indep = zeros(n,1);

% Specify every element in the vector below: 1, 2, ... n
d_depnt__d_indep(1) = 1/A * (Fi - F0);
```

assign1_Q4_part6.m

```
function d_depnt__d_indep = assign1_Q4_part6(indep, depnt)

% Dynamic balance for the CSTR height

%    indep: the independent ODE variable, such as time or length or the reactor
%    depnt: a VECTOR of dependent variables
%
%    Returns:
%
%    d(depnt)
%   ---------- = a vector of ODEs
%    d(indep)

% Assign some variables for convenience of notation: one row per DEPENDENT variable
h = depnt(1);
```

```
% Constants
A = 2.0;    % cross-sectional area [m^2]
Fi = 10.0; % inlet flow rate [m^3/min]
F0s = 10.0;% steady state outlet flow rate [m^3/min]
hs = 2.0;  % (desired) steady state tank height [m]
Kc = 8.0;  % controller gain [m^2/min]

% Other equations
F0 = F0s + Kc*(h - hs);

% Output from this ODE function must be a COLUMN vector, with n rows
% n = how many ODEs in this system?
n = numel(depnt); d_depnt__d_indep = zeros(n,1);

% Specify every element in the vector below: 1, 2, ... n
d_depnt__d_indep(1) = 1/A * (Fi - F0);
```

END