Recall, for least squares models we had decided to mean center the data:

$$
\begin{aligned}
y_i &= b_0 + b_1 x_i \\
\overline{y} &= b_0 + b_1 \overline{x} \\
y_i - \overline{y} &= 0 + b_1 (x_i - \overline{x}) \qquad \text{by subtracting previous lines}
\end{aligned}
$$

- Let $x = x_{\text{raw}} - \text{mean}\,(x_{\text{raw}})$
- Let $y = y_{\text{raw}} - \text{mean}\,(y_{\text{raw}})$
- The model is still the same, except intercept term is forced to zero: $b_0 = 0$

Use the following data:

- $x_{1,\text{raw}} = [1, 3, 6, 8, 11, 13]$, given that mean$(x_1) = 7$
- $x_{2,\text{raw}} = [13, 12, 11, 8, 7, 3]$, given that mean$(x_2) = 9$
- $y_{\text{raw}} = [29, 28, 29, 18, 19, 3]$, given that mean$(y) = 21$

1. Show that the centered vector for $x_1 = [-6, -4, -1, 1, 4, 6]$.
2. Show that the centered vector for $x_2 = [4, 3, 2, -1, -2, -6]$.
3. Show that the centered vector for $y = [8, 7, 8, -3, -2, -18]$.
4. Plot a rough sketch of x_1 vs x_2 (use the centered data)
5. Plot a rough sketch of x_1 vs y (use the centered data)
6. Plot a rough sketch of x_2 vs y (use the centered data)
7. What is your conclusion regarding the relationship of x_1, x_2 and y, using the plots? In the next few questions, match your plots to the numeric values:

8. Calculate $x_1^T x_1 =$
9. Calculate $x_1^T x_2 =$
10. Calculate $x_2^T x_2 =$
11. Calculate $x_1^T y =$
12. Calculate $x_2^T y =$
13. Now form the matrix and vector

$$\mathbf{X} = \begin{bmatrix} -6 & 4 \\ -4 & 3 \\ -1 & 2 \\ 1 & -1 \\ 4 & -2 \\ 6 & -6 \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} 8 \\ 7 \\ 8 \\ -3 \\ -2 \\ -18 \end{bmatrix}$$

and use your answers to the prior questions to calculate:

$$\mathbf{X}^T\mathbf{X} = \begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix} \qquad \mathbf{X}^T\mathbf{y} = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \end{bmatrix}$$

14. Calculate the result of $\mathbf{b} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1} \mathbf{X}^T\mathbf{y}$
15. Compare the signs of $\mathbf{X}^T\mathbf{y}$ to the entries in $\mathbf{b}$.
16. Compare the result to the following R command: `lm(y ~ x1 + x2)`, where `x1`, `x2` and `y` are the vectors above, e.g. `x1 <- c(1, 3, 6, 8, 11, 13)`.