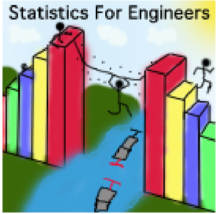
Statistics for Engineers



© Kevin Dunn, 2016

kevin.dunn@mcmaster.ca

http://learnche.mcmaster.ca/

Process Monitoring

Copyright, sharing, and attribution notice

We appreciate:

- if you let us know about **any errors** in the slides
- **any suggestions to improve the notes**

All of the above can be done by writing to

kevin.dunn@mcmaster.ca

or anonymous messages can be sent to Kevin Dunn at

http://learnche.mcmaster.ca/feedback-questions

If reporting errors/updates, please quote the current revision number:

The 5 main uses of data in engineering systems

1. To learn more about a process [e.g. with t-tests, or data visualization]
2. To troubleshoot a system
3. To make predictions from the data
4. To optimize a system proactively [response surface methods]
5. To a monitor a system from (realtime) data

Combine topics 1 and 2 to create a system to visually monitor any process.
The visual plots will help troubleshoot the cause of the problem that is occurring.

Examples of monitoring in everyday situations: medical facilities

[Flickr: 76765412]

Examples of monitoring in everyday situations: control room in a plant

[Flickr: 2311858026]

Examples of monitoring in everyday situations: stock market day-trading

[Flickr: TradingAcademy.com]

Monitoring in engineering scenarios

- Are our product dimensions (or some other production quality measurement) stable?
- How can we quickly detect a slow drift in the process?
- Track the hourly average profit, and process throughput and react to any problems.
- We can show operators the data in an efficient way, so they can move the process away from unsafe operation

Note: process monitoring is mostly **reactive** and not *proactive*. So it is suited to *incremental* process improvement.

AIM: **rapid problem detection**

- then comes diagnosis
- and process adjustment/fixing

Process control has changed over the years

Many of the core concepts are the same, but they are evolving to the new types of data we are seeing now:

- GPS and map/location-based data
- vibrational and acoustic sensor data
- near-infrared and other spectroscopic data
- video and camera digital images

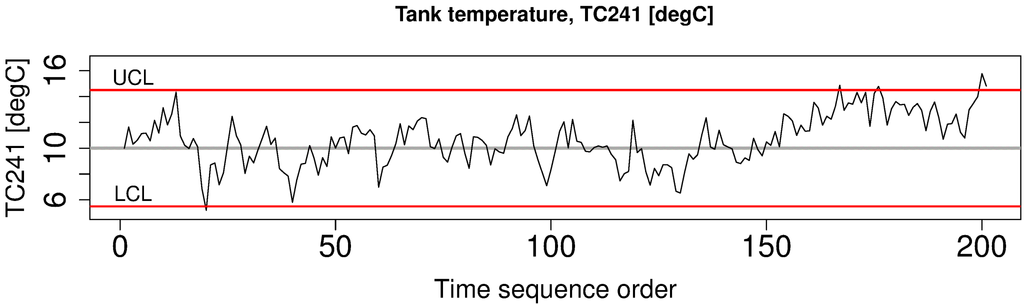
We now have these terms: "data fusion", "big data", "lean manufacturing", "six-sigma", *etc*

To understand what is new, we must understand what has gone on before:

- Marlin, "Process Control", 2nd edition, Chapter 26
- Seborg, Edgar, Mellichamp and Doyle: "Process Dynamics and Control", 3rd edition, Chapter 21

That basic control chart is used to display and detect unusual variability

- it is most often a time-series plot, or sequence plot
- displayed in real-time, or pretty close to real-time
- the vertical axis is the variable being monitored
- a target value (horizontal center-line) should be shown
- one or more limit lines are shown



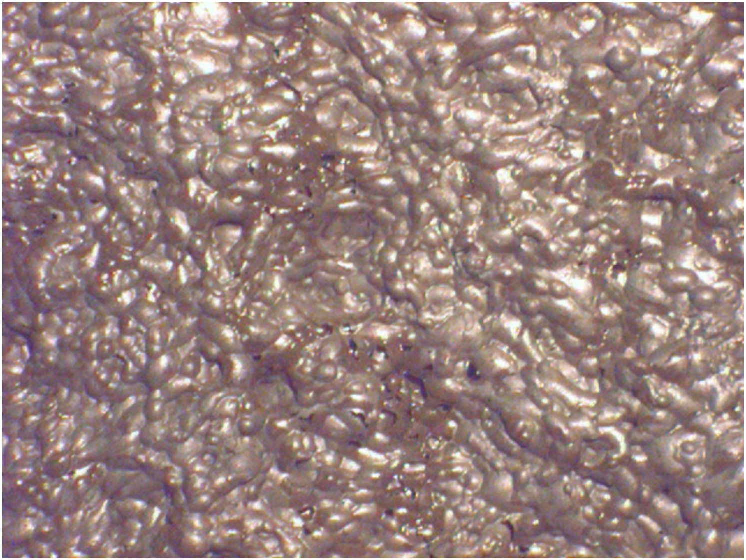
Control charts: froth flotation demonstration

[Illustration from Wikipedia]

[An actual example, ASARCO Mine, Arizona (Flickr: 6850165777)]

Control charts: froth flotation demonstration

The colour and texture of the froth used in the monitoring example:



How to use a monitoring chart after an alarm is raised

- consider the patterns in the plot (called the "signature of the fault/special cause")
- also look at the values in other charts, even if they are in control
- you have to use your engineering knowledge to diagnose the problem's cause

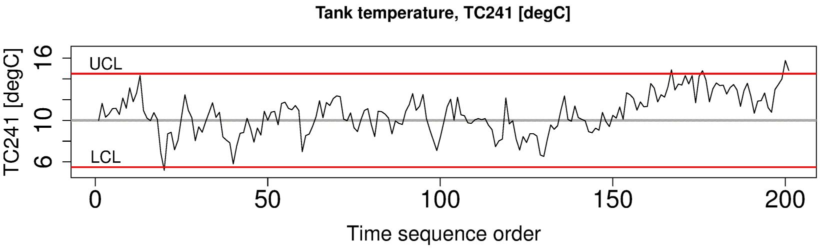
It is probably not wise to create an automatic system that will do the diagnosis. There are many causes that show the same symptoms (signature) in the plots.

Shewhart chart

- Named for *Walter Shewhart* from Bell Telephone and Western Electric, parts manufacturing, 1920's
- A chart for monitoring variable's *location* [the point along the y-axis]

It has:

- a target
- a lower control limit (LCL)
- a upper control limit (UCL)



Being "*in control*" or being "*out of control*"

In a state of statistical control	**A state where we have lost statistical control**
"in control"	"out of control"
process behaviour is stable over time	there are "assignable causes or special causes"
"common cause operation"	destabilizing event(s) occurred
we are on-target	we are off-target
we sell this product to our customers	we need to discount the product, rework it, or scrap it

No action taken if the variable plotted remains within limits.
Do not tamper with the process if you are in control!

What you should monitor on a process is not always obvious

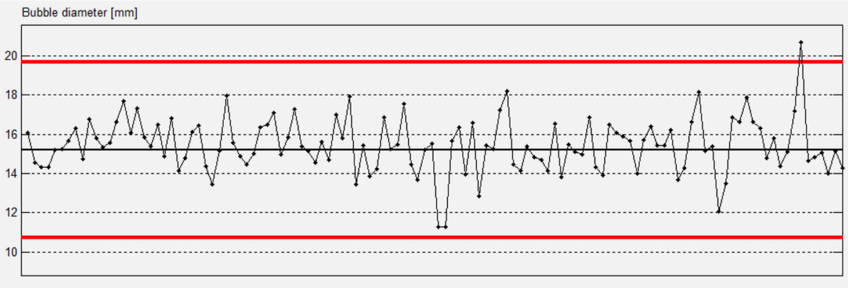
Think of these case studies:

- Waste water treatment process (e.g. anaerobic digester)
- Oil and gas (e.g. a distillation column)
- Food-processing unit (e.g. a fryer)
- Mineral processing plant (e.g. a flotation cell)
- Plastics processing (e.g. a twin-screw extruder)
- Tablet/pharmaceutical manufacturing (e.g. a tablet press)

Once you know which variables to monitor, proceed to "Phase 1" (next video), and then follow that with "Phase 2".

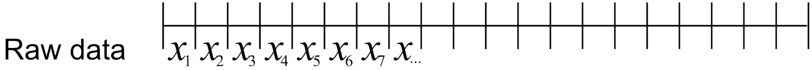
General approach

- First decide what you want to monitor
- **Phase 1**: building and testing from off-line data
  - very iterative: remove outliers
  - calculate limits, test if they are useful, repeat
  - you will spend most of your time here



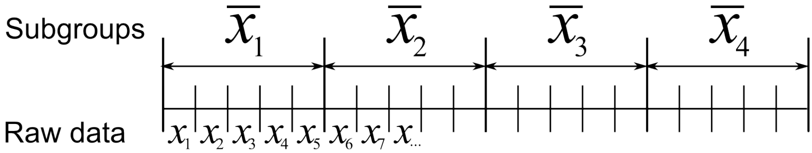
- **Phase 2**: using the control chart
  - on new, unseen data
  - implemented with computer hardware and software
  - usually for real-time display

Shewhart chart: Derivation for the limits (LCL and UCL)



- Take a subgroup of samples, of size n ($n = 5$ in the picture here)
- You are free to select the value of n, the subgroup size
- Calculate $\bar{x}$ from the n values
- What is the distribution of $\bar{x}$?
  - $\bar{x} \sim \mathcal{N}(\mu, \sigma^2/n)$
- Define: $\sigma_{\bar{X}} = \sigma/\sqrt{n} =$ std deviation of my subgroup average
- Assume we know μ (that's the target line) and σ
- $\sigma_{\bar{X}}$ is used to find the LCL and UCL

Shewhart chart: Derivation for the limits (LCL and UCL)



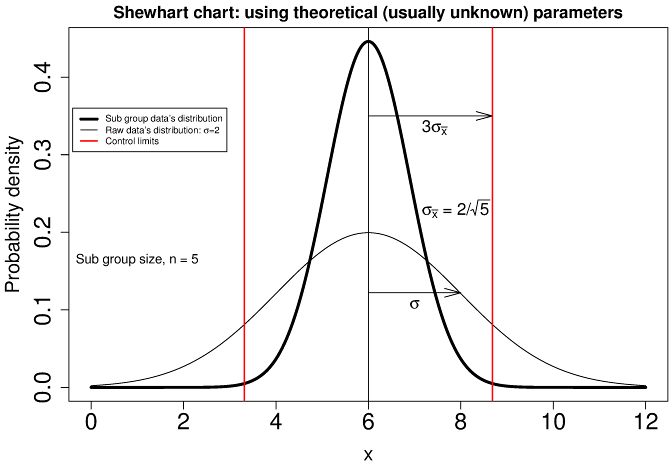
- Take a subgroup of samples, of size n ($n = 5$ in the picture here)
- You are free to select the value of n, the subgroup size
- Calculate $\bar{x}$ from the n values
- What is the distribution of $\bar{x}$?
  - $\bar{x} \sim \mathcal{N}(\mu, \sigma^2/n)$
- Define: $\sigma_{\bar{X}} = \sigma/\sqrt{n}$ = std deviation of my subgroup average
- Assume we know μ (that's the target line) and σ
- $\sigma_{\bar{X}}$ is used to find the LCL and UCL

Shewhart chart: Derivation for the limits (LCL and UCL)

Thin line: raw data have $\mu = 6$ and $\sigma = 2$

Thick line: subgroups have $\sigma_{\overline{x}} = 2/\sqrt{5} = 0.894$, for $n = 5$

- Upper bound $=$ $\mu + 3\sigma_{\overline{x}} = 6 + 3 \times 0.894 = 8.68$
- Lower bound $=$ $\mu - 3\sigma_{\overline{x}} = 6 - 3 \times 0.894 = 3.318$



Shewhart chart: Derivation (theoretical process)

- z-value: $$z = \frac{\bar{x} - \mu}{\sigma_{\bar{X}}}$$
- Confidence interval for μ: $$\bar{x} - c_n \sigma_{\bar{X}} \leq \mu \leq \bar{x} + c_n \sigma_{\bar{X}}$$
- By convention we use $c_n = 3.0$
- Area between LCL and UCL: 99.73%
- A chance of 1 in 370 that a data point, $\bar{x}$, will lie outside these bounds
- The bounds are for $\bar{x}$, not for individual, raw x values

Critical limitation of the above

These calculations assume we know μ and σ

Shewhart chart using estimates, since we don't know the parameters

- For μ: or use the **median** of a long sequence of good operating data
- For μ: or just use known target value
- For μ: use $\overline{\overline{x}} = \dfrac{1}{K}\displaystyle\sum_{k=1}^{K} \overline{x}_k$; where K = number of phase 1 groups
- For σ:
  - Define: s_k = standard deviation of n values
  - Define: $\overline{S} = \dfrac{1}{K}\displaystyle\sum_{k=1}^{K} s_k$
  - Overall standard deviation is estimated from $\hat{\sigma} = \dfrac{\overline{S}}{a_n}$, where a_n is a correction factor

n	2	3	4	5	6	7	8
a_n	0.798	0.886	0.921	0.940	0.952	0.959	0.965

$$\text{limits} = \overline{\overline{x}} \pm 3 \cdot \frac{\sigma}{\sqrt{n}} \qquad \text{LCL} = \overline{\overline{x}} - 3 \cdot \frac{\overline{S}}{a_n\sqrt{n}} \qquad \text{UCL} = \overline{\overline{x}} + 3 \cdot \frac{\overline{S}}{a_n\sqrt{n}}$$

Example: we measure colour values on a rubber product

- e.g. given these 5 colour values: [231, 251, 235, 241, 227]
  - $\bar{x}_1 = 237$ would be our data point for the Shewhart chart
  - $s_1 = 9.38$ is the standard deviation within the subgroup
- e.g. then if these 5 colour values were recorded next: [252, 253, 247, 232, 244]
  - $\bar{x}_2 = 245.6$ and $s_2 = 8.44$

That was for practice.

Here are the actual raw colour values for you try building a Shewhart chart with:
[252, 252, 230, 249, 242, 238, 245, 224, 242, 244, 236, 245, 252, 228, 236, 252, 254, 224, 236, 239, 251, 224, 254, 253, 222, 237, 233, 251, 235, 249, 245, 248, 233, 243, 220, 249, 220, 227, 251, 241, 233, 235, 221, 254, 235, 253, 251, 242, 253, 241, 231, 232, 233, 247, 221, 246, 243, 225, 229, 238, 243, 254, 246, 239, 249, 254, 255, 253, 252, 251, 221, 224, 227, 236, 226, 245, 220, 233, 238, 220, 240, 225, 232, 242, 247, 239, 228, 223, 222, 230, 243, 220, 227, 252, 252, 245, 231, 238, 246, 241]

Download the data and try it for yourself: http://yint.org/rubber

Example: we measure colour values on a rubber product

Using the raw data you should find the 20 $\bar{x}_k$ values to be

```
[245, 239, 239, 241, 241, 241, 238, 238, 236, 248,
 233, 236, 246, 253, 227, 231, 237, 228, 239, 240]
```

and also that $\bar{\bar{x}} = 238.8$ and $\bar{S} = 9.28$

Phase 1 workflow: *build* the monitoring chart

- Calculate: LCL and UCL
- Any $\bar{x}$ points outside limits?
- If so, exclude these outliers and recalculate limits.

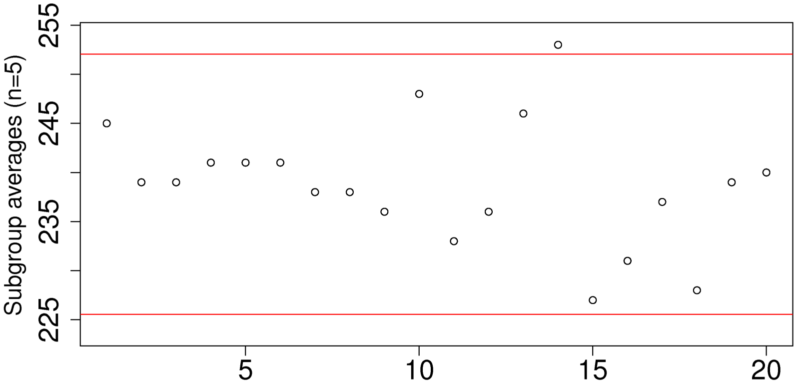
Phase 2 workflow: *using* the monitoring chart

- Obtain n new data points
- Calculate the subgroup average, $\bar{x}$
- Add this $\bar{x}$ point to the plot

Download the data and try it for yourself: http://yint.org/rubber

Example: we measure colour values on a rubber product

$$\text{LCL} = 238.8 - 3 \cdot \frac{9.28}{(0.94)(\sqrt{5})} = 225.6 \text{ and UCL} = 238.8 + 3 \cdot \frac{9.28}{(0.94)(\sqrt{5})} = 252.0$$



Sample with value of $\bar{x}_i = 253$ exceeds these limits. After excluding it:

- new $\bar{\bar{x}} = 238.0$, and new $\bar{S} = 9.68$
- new LCL $= 224$, new UCL $= 252$

Back to the bigger picture

We have covered this in this video:

- **Phase 0**: decide what to monitor
- **Phase 1**: building and testing from off-line data
  - very iterative: remove outliers
  - calculate limits, test if they are useful, repeat
  - you will spend most of your time here

We are here now:

- **Phase 2**: using the control chart
  - on new, unseen data
  - implemented with computer hardware and software
  - usually for real-time display

Coming up ...

Quantifying the chart's performance is important: *you will need these numbers to convince your manager that it is worth spending money on implementing these charts*.

We also have to validate how frequently false alarms occur, and some other interesting metrics.

Our goal with this video

Quantifying performance of a control chart:

- contrast process control with process monitoring
- type I errors
- type II errors
- what happens to these errors when we adjust the limits

What this topic is important:

- We know that quality is not optional; customers move onto suppliers that provide quality products
- Quality is generally not a cost-benefit trade-off
  - Customer's value quality; but it is a long-term benefit
  - Example: car sales in North America: the steady rise of the non-North America manufacturers

"Process monitoring" contrasted to "feedback control"

You will hear "Statistical Process Control" (SPC) $\equiv$ "process monitoring"

You will hear the term "control chart" $\equiv$ monitoring chart

Process control

- feedback continuously applied
- **check** for **deviations**
- take action automatically and **frequently**

Process monitoring

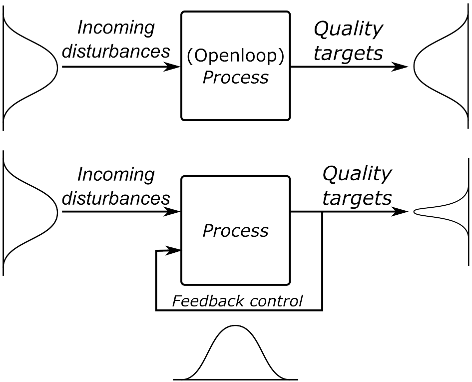
- a continuous chart is shown
- **monitor** for **special causes**
- adjustments are **infrequent** and manual

Summary:

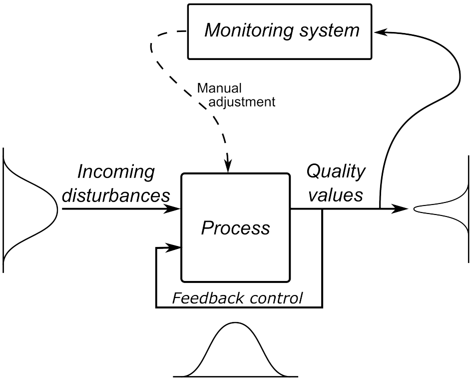
- Process monitoring: make *permanent* adjustments to reduce variability
- Feedback control: *temporarily* compensates for the problem all the time

How should a control chart be used?

- watch for signatures that identify a root cause
- eliminate the root cause, where possible, so the problem never comes back



Ideally: feedback is never added; we simply ensure no disturbance ever happens to our process.



In practice: we need feedback to react fast; and monitoring to identify when quality drifts outside the control limits

Now on to judging the chart's performance: error probability

The $\pm 3\sigma_{\overline{x}}$ limits have a reason:

- If the $\overline{x}_i$ values are normally distributed (they should be, due to the CLT)
- then, 1 point in 370 will lie outside the limits by chance (99.73% inside limits)
- *false alarm rate*, denoted as $\alpha = 0.0027$
- *Synonyms*:
  - producer's risk (acceptance sampling)
  - false positive (diseases)
  - false reject rate
- Statistics: called a **Type I error**

Type I error: $\overline{x}_i$ is typical of normal operation, but falls outside UCL or LCL limits

Operators hate chasing down a false alarm. You do not get many chances raising a false-alarm. You will be ignored after your chart raises a couple of these.

Now on to judging the chart's performance: error probability

There are another type of error we can make: *false negative*

Type II error: $\overline{x}_i$ is not from normal operation, but falls inside UCL or LCL limits

- *false negative*, denoted as β
- *Synonyms*:
  - consumer's risk (acceptance sampling)
  - false negative (diseases)
  - false acceptance rate

Now on to judging the chart's performance: error probability

In pseudo-math:

$\alpha = Pr\,(\overline{x}$ is in control, but lies outside the limits$)$	Type I (false alarm)
$\beta = Pr\,(\overline{x}$ is not in control, but lies inside the limits$)$	Type II (false negative)

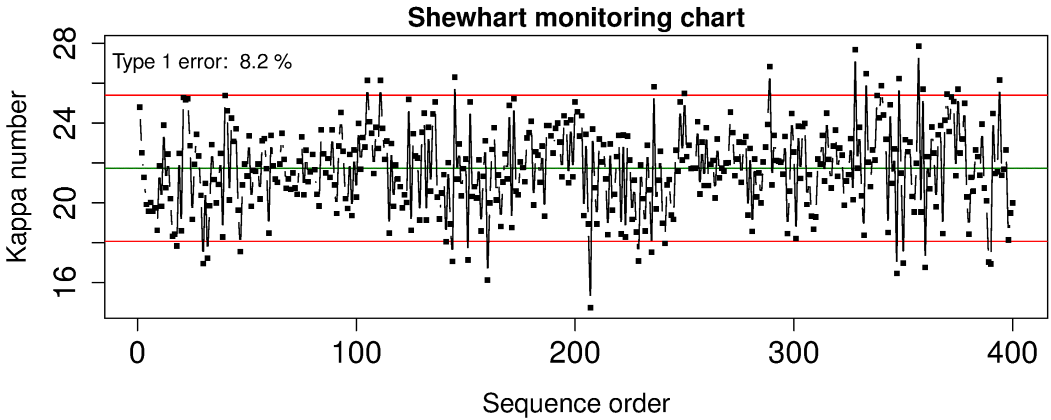
Notice the asymmetry in type I (false positive) and II (false negative) errors in these cases

- disease diagnosis [consider that you have the disease]
- airport screening for weapons [consider the passenger's point of view; consider the smuggler's point of view]
- trial by jury [consider the defendant's point of view]

Adjusting the chart's performance to get the desired false alarm rate

Control chart limits are not set in stone. Adjust them!

- α to high? Move LCL lower and UCL up [not incorrect to do this!]

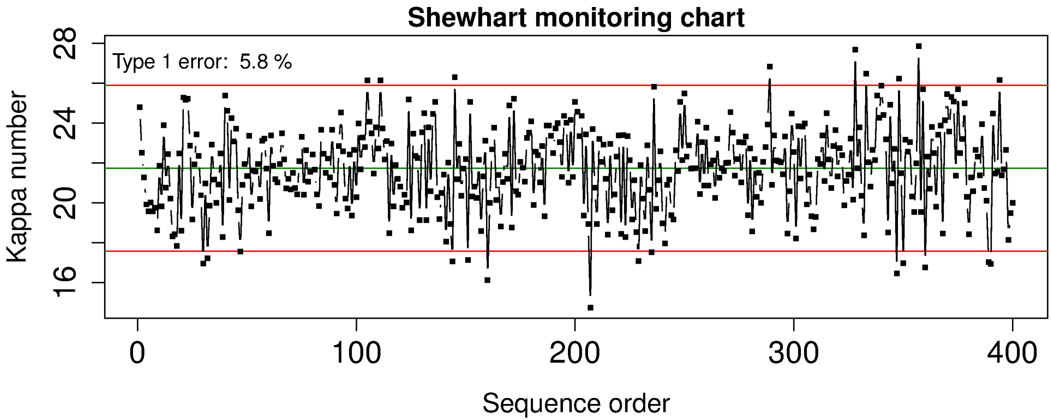


As you increase UCL, $\alpha \rightarrow 0$, but $\beta \rightarrow 1$

Adjusting the chart's performance to get the desired false alarm rate

Control chart limits are not set in stone. Adjust them!

- α to high? Move LCL lower and UCL up [not incorrect to do this!]

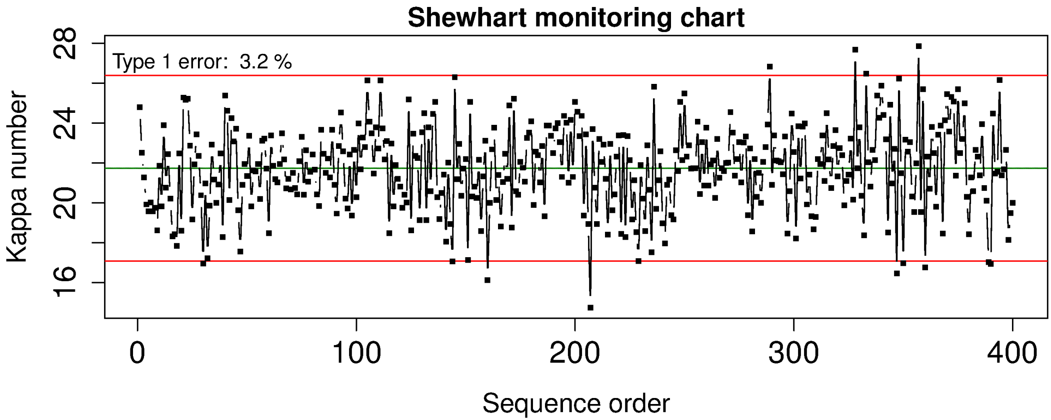


As you increase UCL, $\alpha \to 0$, but $\beta \to 1$

Adjusting the chart's performance to get the desired false alarm rate

Control chart limits are not set in stone. Adjust them!

- α to high? Move LCL lower and UCL up [not incorrect to do this!]

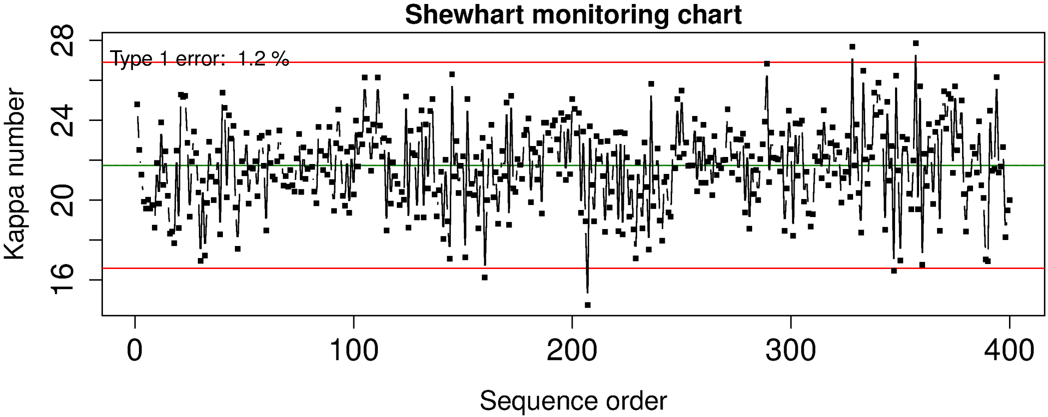


As you increase UCL, $\alpha \to 0$, but $\beta \to 1$

Adjusting the chart's performance to get the desired false alarm rate

Control chart limits are not set in stone. Adjust them!

- α to high? Move LCL lower and UCL up [not incorrect to do this!]

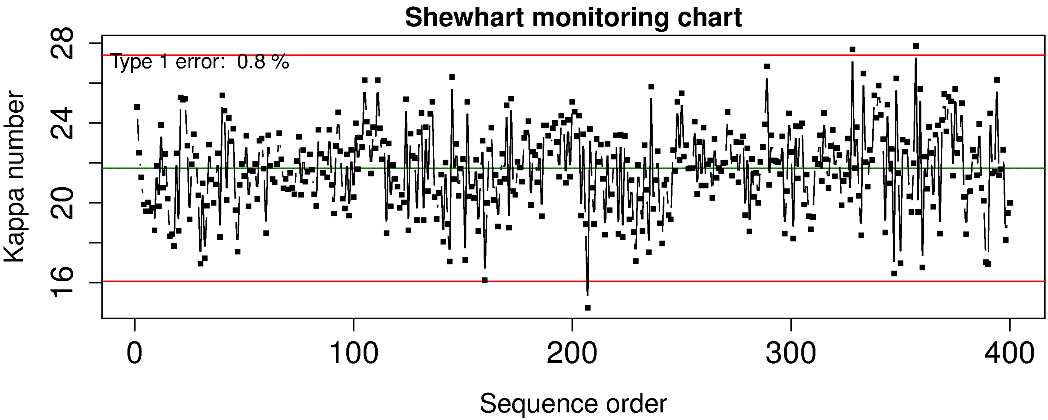


As you increase UCL, $\alpha \to 0$, but $\beta \to 1$

Adjusting the chart's performance to get the desired false alarm rate

Control chart limits are not set in stone. Adjust them!

- α to high? Move LCL lower and UCL up [not incorrect to do this!]

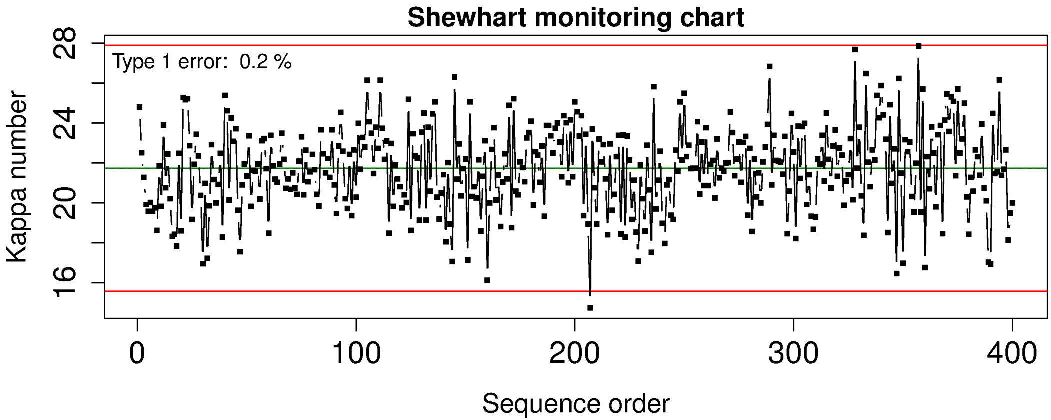


As you increase UCL, $\alpha \to 0$, but $\beta \to 1$

Adjusting the chart's performance to get the desired false alarm rate

Control chart limits are not set in stone. Adjust them!

- α to high? Move LCL lower and UCL up [not incorrect to do this!]



As you increase UCL, $\alpha \to 0$, but $\beta \to 1$

Extensions: Western Electric Rules

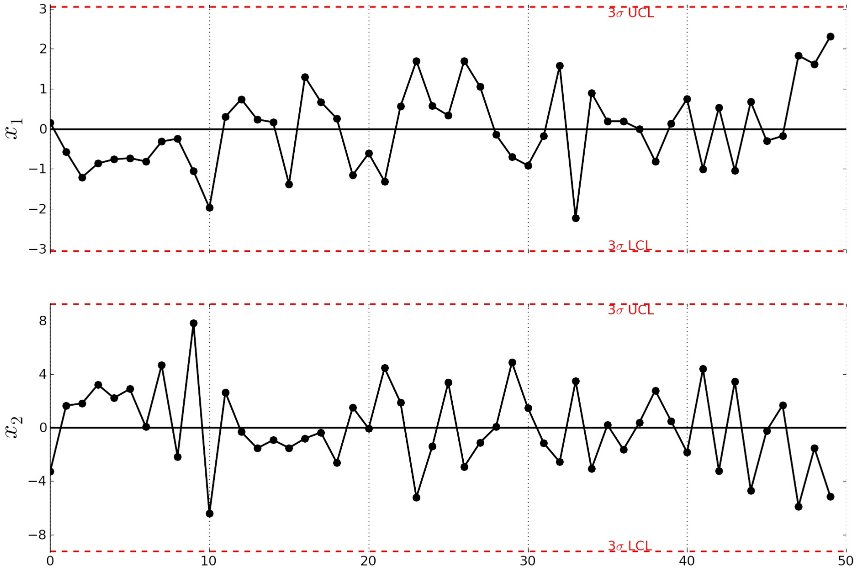
- Basic Shewhart chart is not too sensitive to process shifts. So supplement control limits with these additional heuristic "rules". Raise an alarm when:
  - 2 out of 3 points lie beyond 2σ on the same side of the target
  - 4 out of 5 points lie beyond 1σ on the same side of the target
  - 8 successive points lie on the same side of the center line

The theoretical ARL is reduced by using these rules

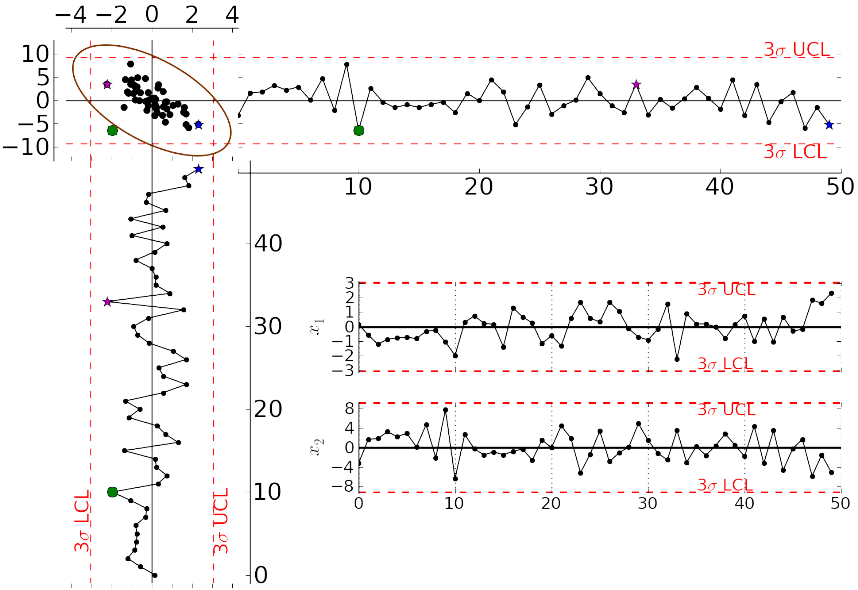
- **Adding robustness**: use robust methods in phase 1: see notes for a journal reference.

- **Warning limits**:
  - warning at $\pm 2\sigma$ (orange coloured lines or background)
  - action at $\pm 3\sigma$ (red coloured lines or background)

What is the relationship between these variables?



Monitoring correlated variables: we need a better tool



CUSUM (**cu**mulative **sum**) charts

- Shewhart chart takes a long time to detect shift in the mean, away from target, T
- CUSUM formula:

$$\begin{aligned} S_0 &= (x_0 - T) \\ S_1 &= (x_0 - T) + (x_1 - T) = S_0 + (x_1 - T) \\ S_2 &= (x_0 - T) + (x_1 - T) + (x_2 - T) = S_1 + (x_2 - T) \end{aligned}$$

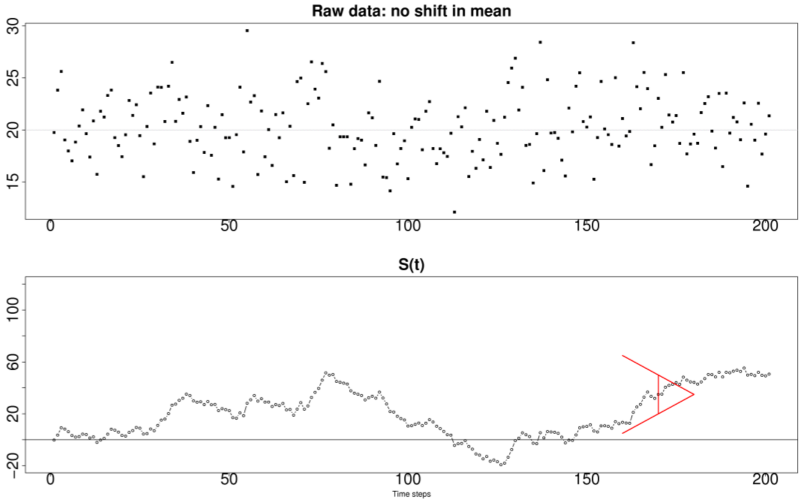
CUSUM formula:

$$S_t = S_{t-1} + (x_t - T)$$

- When a process shifts by Δ, then we are adding Δ to every x_t
- This accumulates: creates a steep up or down slope

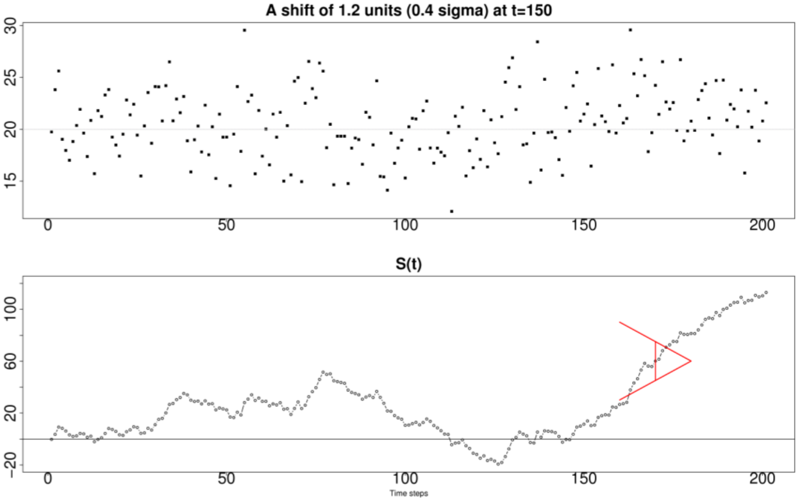
CUSUM charts: everything is OK, “in-control”

- $\mu = 20$ and $\sigma = 3$



CUSUM charts: "out-of-control"

- Shift of 1.2 units $(1.2/3 = 0.4\sigma)$ starts at $t = 150$, caught at around $t = 180$; far earlier than a Shewhart chart



Using a CUSUM chart

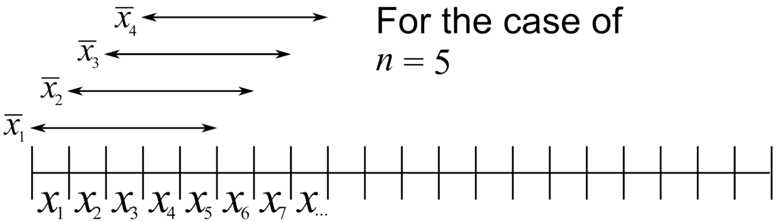
- Type I and II error set by the angle and distance of the V-mask
- Implemented by computers
- If a fault is detected, reset S_t to a new value and restart chart

CUSUM charts:

- should be used when we require very sensitive monitoring around a **specified, fixed target**

EWMA charts: what is a moving average?

1. Shewhart: each subgroup is independent (unrelated), no "memory"
2. CUSUM: infinite build up of errors, all the way back to $t = 0$
3. Moving average (MA) chart: has a "window" of memory:



$$
\begin{aligned}
\bar{x}_t &= \frac{1}{n}x_{t-1} + \frac{1}{n}x_{t-2} + \ldots + \frac{1}{n}x_{t-n} \\
\bar{x}_t &= 0.25x_{t-1} + 0.25x_{t-2} + 0.25x_{t-3} + 0.25x_{t-4} + 0 \qquad \text{for } n = 4
\end{aligned}
$$

- essentially gives equal "weight" of $\dfrac{1}{n}$ to every raw data point

EWMA derivation example

- Exponentially Weighted Moving Average = EWMA
  - don't give equal weights, rather ...
  - heavier weights for the most recent observations
  - small weights further back in time

	$\hat{x}_{t+1} =$	$\lambda x_t +$	$\lambda(1-\lambda)x_{t-1} +$	$\lambda(1-\lambda)^2 x_{t-2} +$	$\lambda(1-\lambda)^3 x_{t-3} + \ldots$
$\lambda = 0.8:$	$\hat{x}_{t+1} =$	$0.8x_t +$	$0.16x_{t-1} +$	$0.032x_{t-2} +$	$0.006x_{t-3} + \ldots$
$\lambda = 0.6:$	$\hat{x}_{t+1} =$	$0.6x_t +$	$0.24x_{t-1} +$	$0.096x_{t-2} +$	$0.0384x_{t-3} + \ldots$
$\lambda = 0.2:$	$\hat{x}_{t+1} =$	$0.2x_t +$	$0.16x_{t-1} +$	$0.128x_{t-2} +$	$0.1024x_{t-3} + \ldots$
MA chart*	$\hat{x}_{t+1} =$	$0.25x_t +$	$0.25x_{t-1} +$	$0.25x_{t-2} +$	$0.25x_{t-3}$

* MA chart with 4 samples per group: weight equals $1/4 = 0.25$

- As $\lambda \to 0$: smoother chart, uses more history, less current data
- As $\lambda \to 1$: chart uses more current data

EWMA derivation

We would rather use a recursive formula though (so we don't have to store all the raw x_t values in memory)

$$
\begin{aligned}
x_t &= \text{raw data value at time step } t \\
\hat{x}_{t+1} &= \hat{x}_t + \lambda e_t \\
e_t &= x_t - \hat{x}_t
\end{aligned}
$$

$$\hat{x}_{t+1} = \hat{x}_t + \lambda e_t = \hat{x}_t + \lambda(x_t - \hat{x}_t)$$

To start if off:

- $\hat{x}_0 = T$
- $e_0 = 0$
- $\hat{x}_1 = T$

And: $0 \leq \lambda \leq 1$

Only store x_t and $\hat{x}_t$ to calculate x_{t+1}

EWMA derivation: an alternative interpretation

$$\begin{aligned} x_t &= \text{raw data value at time step } t \\ \hat{x}_{t+1} &= \hat{x}_t + \lambda e_t \\ e_t &= x_t - \hat{x}_t \end{aligned}$$

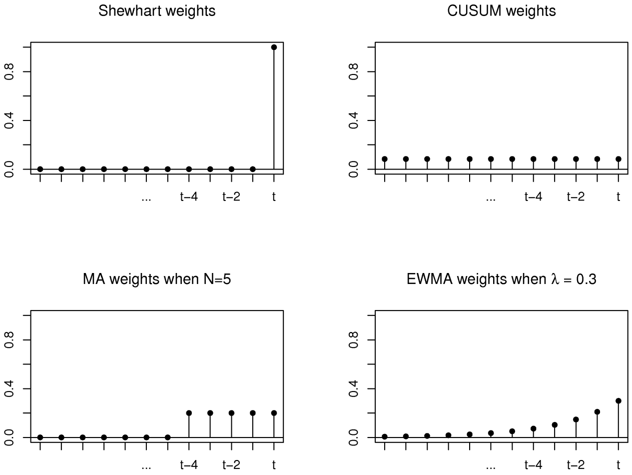
Substitute in the e_t :

$$\begin{aligned} \hat{x}_{t+1} &= \hat{x}_t + \lambda \left(x_t - \hat{x}_t \right) \\ \hat{x}_{t+1} &= \lambda x_t + \left(1 - \lambda \right) \hat{x}_t \end{aligned}$$

- Shows that EWMA is a one-step ahead predictor for $\hat{x}_{t+1}$
  1. x_t: current point, weighted by λ
  2. $\hat{x}_t$: historical data, weighted by $(1 - \lambda)$

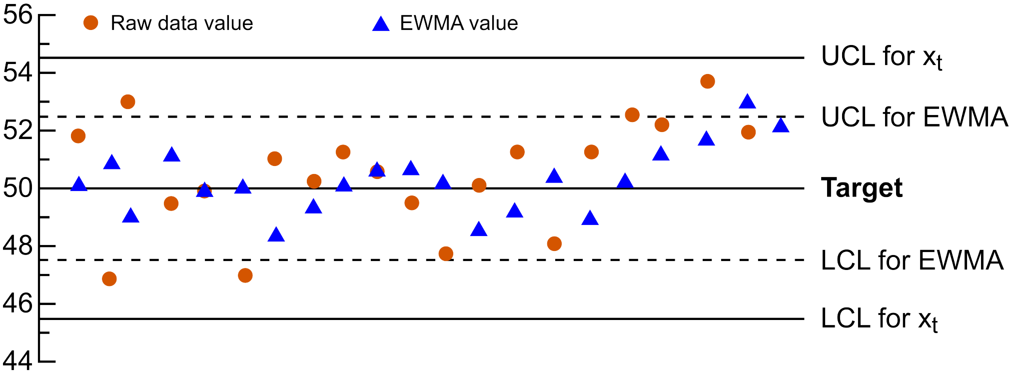
- As $\lambda \to 0$: smoother chart, uses more history, less current data (more CUSUM like)
- As $\lambda \to 1$: chart uses more current data (Shewhart-like)

EWMA derivation example



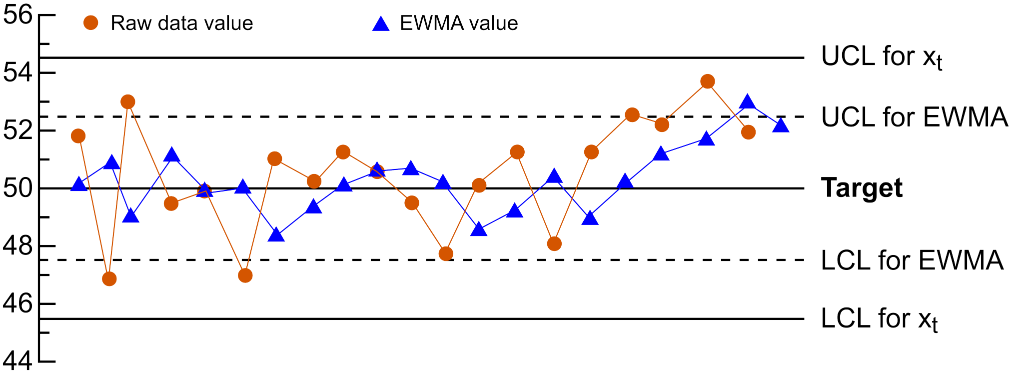
- As $\lambda \to 0$: smoother chart, uses more history, less current data
- As $\lambda \to 1$: chart uses more current data (Shewhart-like)

EWMA example



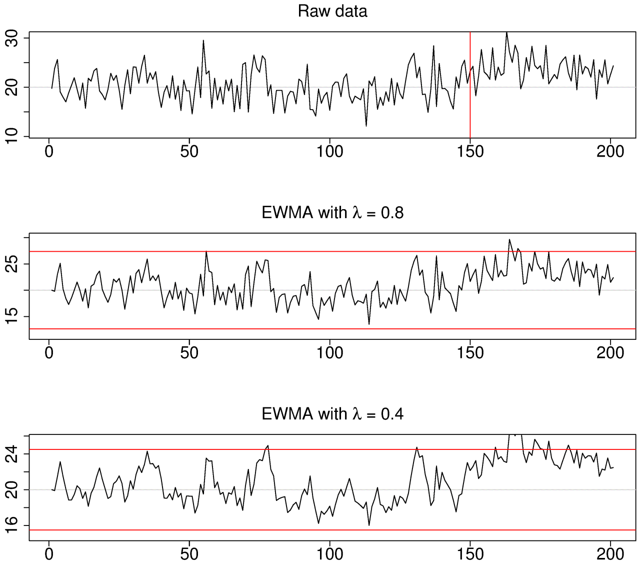
From: Hunter, "The Exponentially Weighted Moving Average", *Journal of Quality Technology*, **18**, p 203-210, 1986.

EWMA example

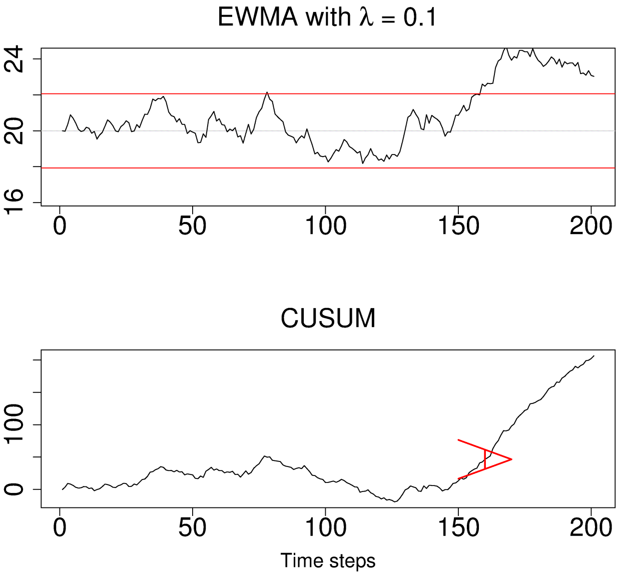


From: Hunter, "The Exponentially Weighted Moving Average", *Journal of Quality Technology*, **18**, p 203-210, 1986.

EWMA visual example (fault occurs at $t = 150$)



EWMA visual example (fault occurs at $t = 150$)



EWMA limits

$$\begin{aligned} \text{LCL} &= \bar{\bar{x}} - 3 \cdot \sigma_{\text{Shewhart}} \sqrt{\frac{\lambda}{2-\lambda}} \\ \text{UCL} &= \bar{\bar{x}} + 3 \cdot \sigma_{\text{Shewhart}} \sqrt{\frac{\lambda}{2-\lambda}} \end{aligned}$$

- σ_{Shewhart}: standard deviation from Shewhart chart $= \dfrac{\overline{S}}{a_n\sqrt{n}}$

Nice implementation: show both Shewhart and EWMA on the same chart

- This gives you the usual Shewhart monitoring, but for a
- slow-moving processes with long gaps between samples,
- the EWMA helpfully also gives a one-step ahead prediction

Other charts

- The *S chart*: monitor variance
- The *R chart*: precursor to the *S chart* (not common anymore)
- *np chart* and *p chart*: monitoring proportions of pass/fail or good/bad ratings
- *Exponentially weight moving variance* (EWMV): used for monitoring product variability

What should we monitor?

Recall the aim is to **react early** to bad, or unusual operation:

- implies monitoring variables in near real-time
- laboratory measurements are good, but take longer to acquire
- don't wait for bad production to be over, catch it early

Key points

- Apply monitoring at every step in the manufacturing line/system
- Obtain low variability early on; don't wait to the end

Problem isn't how to monitor, rather, what do we monitor?

What should we monitor?

Measurements from real-time systems are:

- available more frequently (less delay) than lab measurements
- often are more precise
- more meaningful to the operating staff
- contains “fingerprint” of problem (helps for diagnosis)

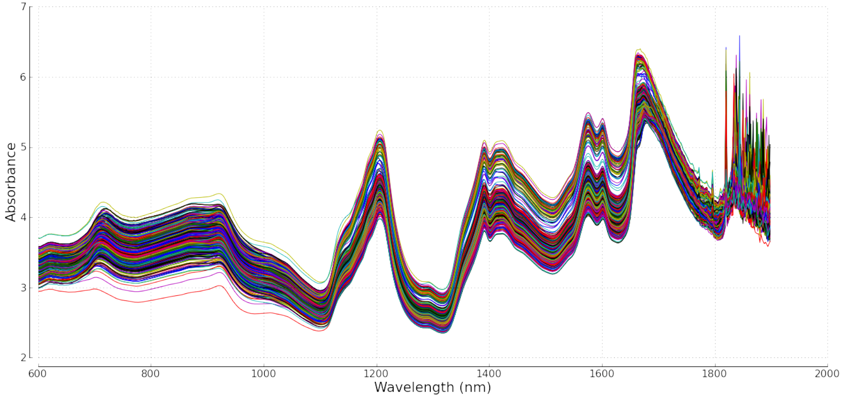
Variables don’t need to be from on-line sensors: could also be a calculation

Lab measurements have long time delay:

- process already shifted by the time lab values detect a problem
- harder to find cause-and-effect for diagnosis

Monitoring in today's context

We don't measure a single number in many cases:



460 spectra (lines) measured at 650 evenly spaced wavelengths (x-axis). The y-axis is the absorbance at each wavelength.

What can we monitor with this data?

Monitoring with image data

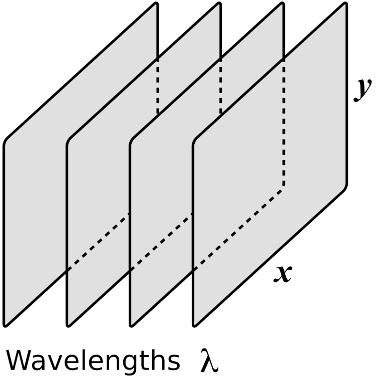


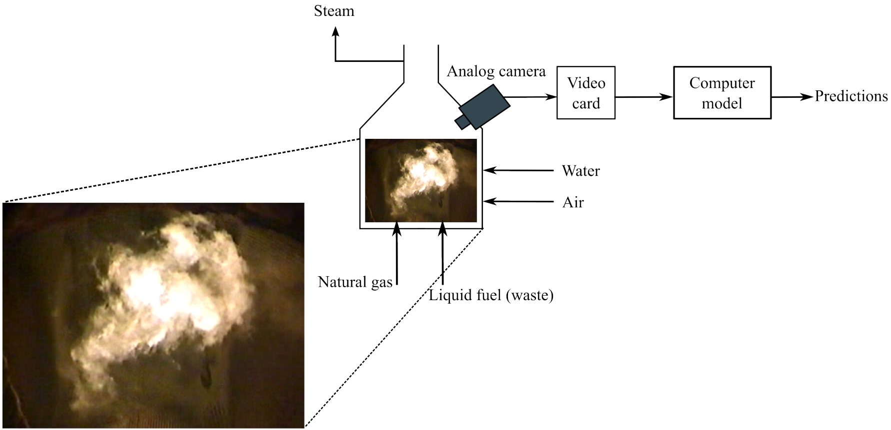
Image data: very common situation now

- many easy measurements for liquids and gases;
- but for solids: use image data
- medical imaging

The wavelength dimension (spectral dimension):

- 1 channel: grayscale
- 3 channels: colour image, RGB image
- multiple channels: hyperspectral image (e.g. NIR camera)

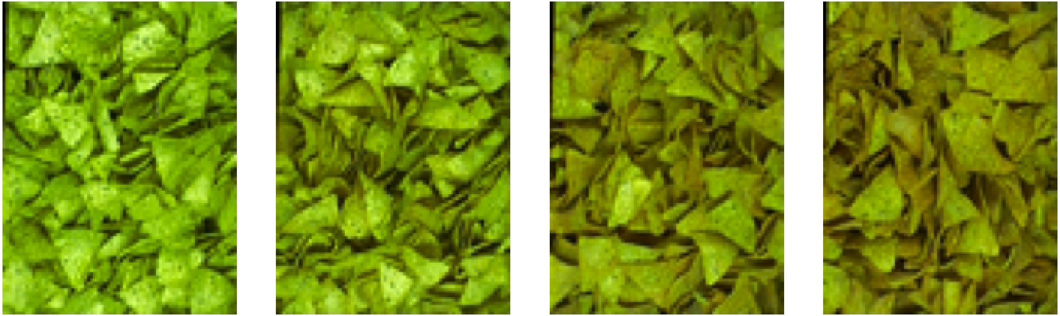
Flame monitoring application



- Liquid waste stream has variable energy content
- How can we maintain a desired steam flow from the boiler when the heating source is varying?

Seasoning application

We can see a change in product appearance with more seasoning: [1]

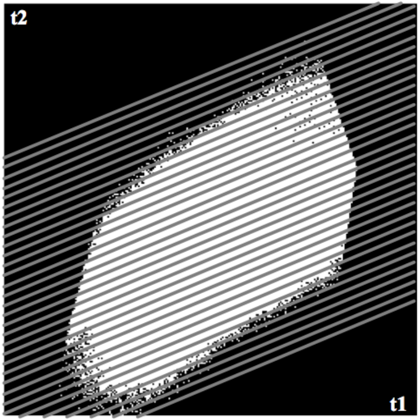


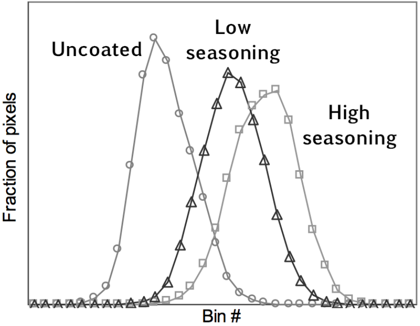
Seasoning levels increase when going from left to right $\longrightarrow$

[1] From Honglu Yu's PhD (used with permission)

Feature extraction in the score space (phase 1)

Create bins in the score space (eigen-decomposition of the raw image data); then count pixels:





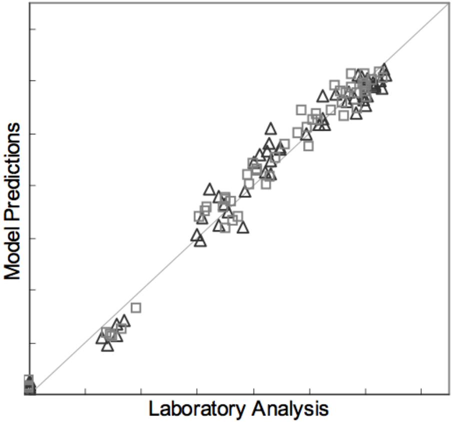
Direction of bins: direction in which seasoning level shifts the score plot [2]

[2] Other binning methods described in journal publications and thesis

Model building and predictions (phase 1)

PLS predictive model:

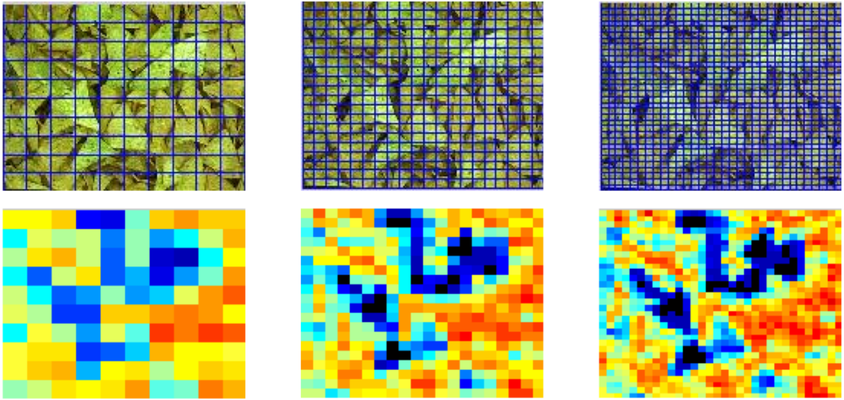
- $\mathbf{X}$-space: cumulative histogram of *fractional* bin counts
  - Note the $\mathbf{X}$-space will be very collinear



triangles = training data; squares = testing data

Apply the predictive model to new images (phase 2)

- Apply procedure to subset of pixels:



Monitoring system built for:

- average seasoning level (also placed under automatic feedback control)
- seasoning *variance*

Monitoring in industrial practice

- Widely used in industry, at all levels
- Management: monitor plants, geographic region, countries (e.g. hourly sales by region)
  - Dashboards, ERP, BI, KPI

- Challenges for you:
  - Getting the data out
  - Real-time use of the data (value of data decays exponentially)
  - Training is time consuming
  - Bandwidth/network/storage

General workflow [1]

1. Identify variable(s) to monitor.
2. Retrieve historical data (computer systems, or lab data, or paper records)
3. Import data and just plot it.
   - Any time trends, outliers, spikes, missing data gaps?
4. Locate regions of stable, common-cause operation.
   - Remove spikes and outliers
   - Cleaned data is your phase 1 data
5. Split phase 1 data into a 60% and 40% split.
   - The 60% split is for calculating model limits (phase 1)
   - The 40% is for testing later on (phase 2).
6. Keep outlier data as a separate testing set: to validate detection
7. Calculate control limits (UCL, LCL), on the 60% data chunk
8. Test your chart on **new, unused** data. *How does my chart work?*
   - Quantify type I error on cleaned 40% chunk
   - Quantify type II error on outlier data

General workflow [2]

9. Adjust the limits
10. Repeat these steps, as needed to achieve levels of error
11. Run chart on your desktop computer for a couple of days (phase 2)
    - Confirm unusual events with operators; would they have reacted to it? False alarm?
    - Refine your limits (back to phase 1)
12. Not an expert system - will not diagnose problems:
    - use your engineering judgement; look at patterns; knowledge of other process events; *troubleshooting* skills!
13. Demonstrate to your colleagues and manager
    - But have with dollar values for them: quantify the value of the chart to them
    - e.g. "this chart would have saved $50,900 of bad quality product being made"
14. Installation and operator training will take time
15. Listen to your operators
    - make plots interactive - click on unusual point, it drills-down to give more context

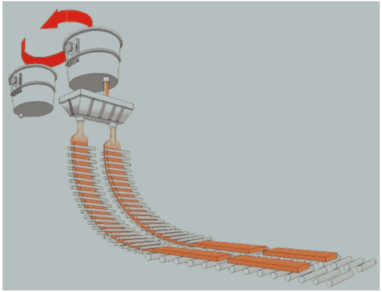
Industrial case study: Dofasco

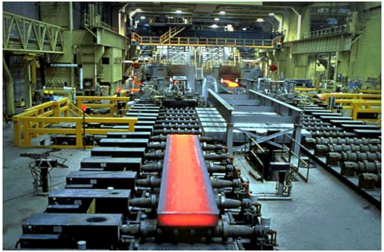
- ArcelorMittal in Hamilton (formerly called Dofasco) has used multivariate process monitoring tools since 1990's
- Over 100 applications used daily
- Most well known is their casting monitoring application, Caster SOS (Stable Operation Supervisor)
- It is a multivariate monitoring system

Dofasco case study: slabs of steel

All screenshots with permission of Dr. John MacGregor

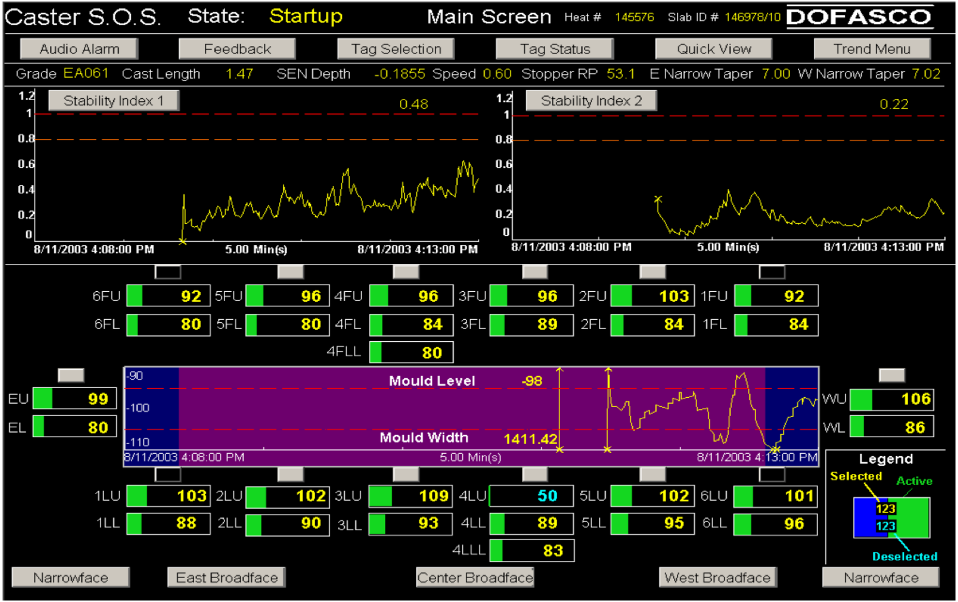
Dofasco case study: casting



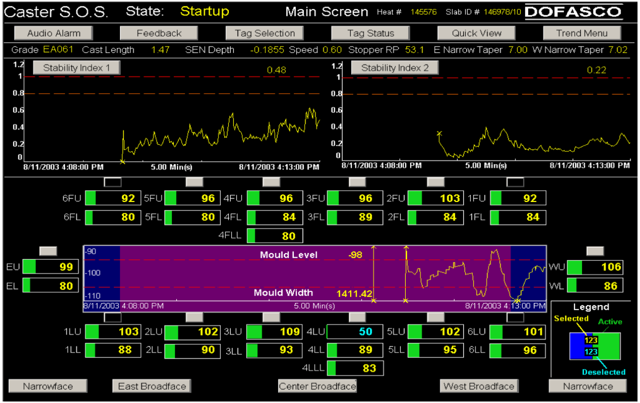


Dofasco case study: breakout

Dofasco case study: monitoring for breakouts

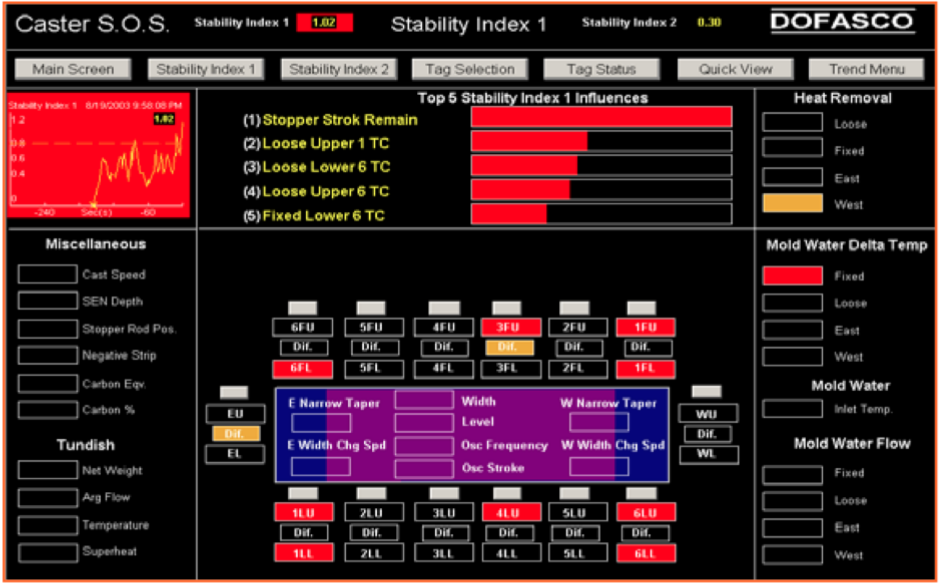


Dofasco case study: monitoring for breakouts

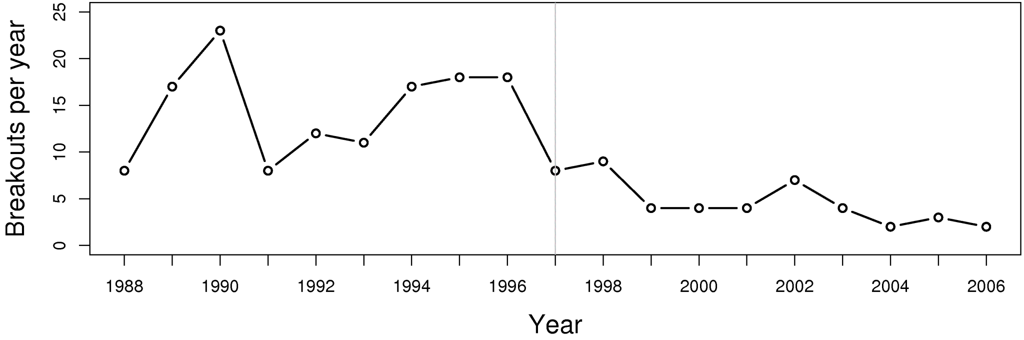


- Stability Index 1 and 2: one-sided monitoring chart
- Warning limits and the action limits.
- A two-sided chart in the middle
- Plenty of other operator-relevant information

Dofasco case study: an alarm



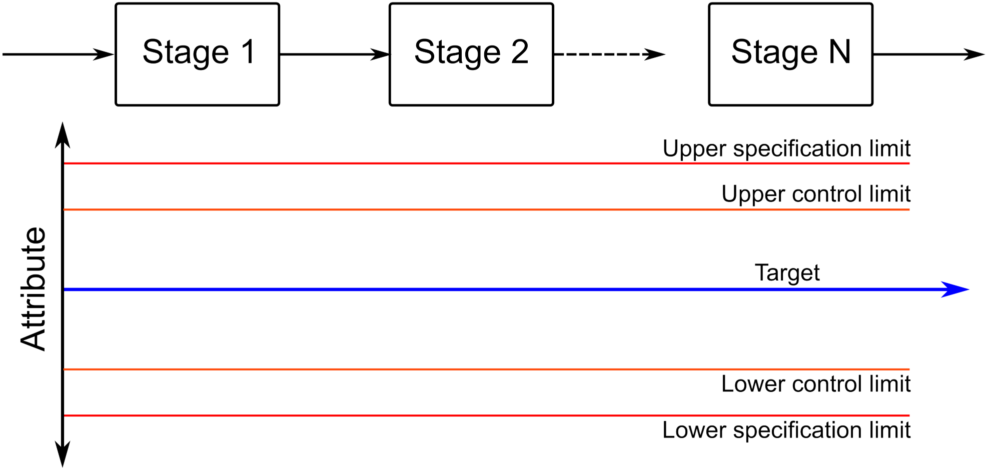
Dofasco case study: economics of monitoring



- Implemented system in 1997; multiple upgrades since then
- Economic savings: more than $ 1 million/year
  - each breakout costs around $200,000 to $500,000
  - process shutdowns and/or equipment damage
  - more than justifies the costs and person-hours to implement the system

Process capability: centered process

Process capability ratio (PCR) can be calculated for any attribute, mainly used for attributes from the final stage though.



Process capability defined for a centered process

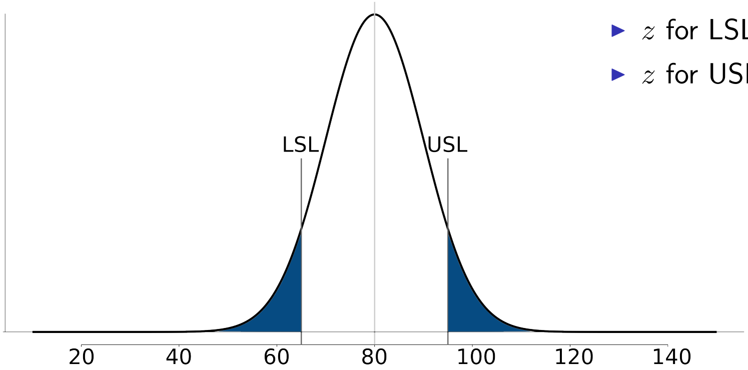
$$\text{PCR} = \text{C}_\text{p} = \frac{\text{Upper specification limit} - \text{Lower specification limit}}{6\sigma}$$

- Use an estimate for σ
- Estimate this σ from in-control operation (i.e. the process is stable)
- To interpret C_p we assume the attribute is centered between the LSL and USL
- Assumes the attribute has normal distribution: check with `qqPlot(...)`

Note: the specification limits are set by your customers, or from internal company criteria.

Interpreting and using the PCR value: by example

Let mean=80, LSL=65, USL=95 and $\hat{\sigma} = 10$. You can confirm that PCR = 0.5

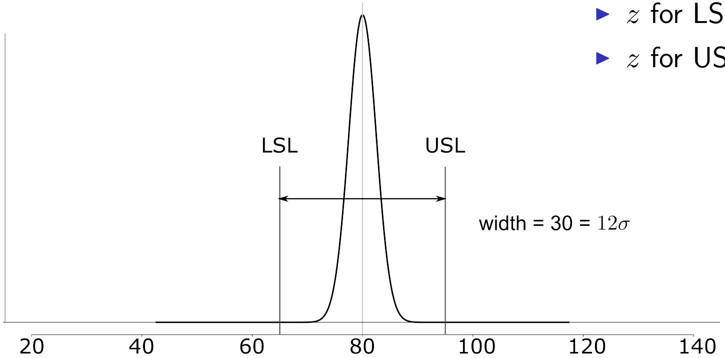


- z for LSL $= (65 - 80)/10 = -1.5$
- z for USL $= (95 - 80)/10 = +1.5$

Shaded area probability = `pnorm(-1.5) + (1-pnorm(1.5))` = 13.4%

Interpreting and using the PCR value: by example

Let mean=80, LSL=65, USL=95 and $\hat{\sigma} = 2.5$. You can confirm that PCR = 2.0

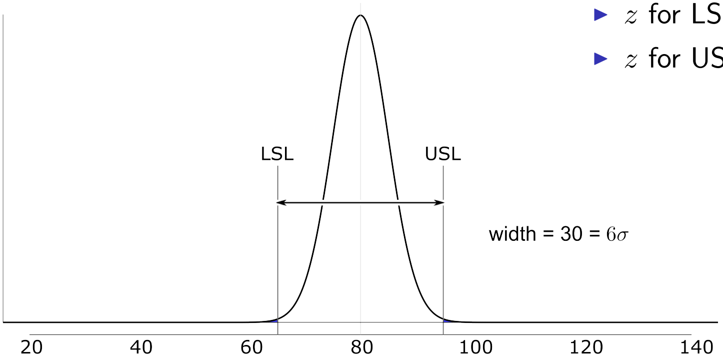


- z for LSL $= (65 - 80)/2.5 = -6$
- z for USL $= (95 - 80)/2.5 = +6$

Shaded area probability $= 1.973 \times 10^{-9} \times 100 \approx 0$

Interpreting and using the PCR value: by example

Let mean=80, LSL=65, USL=95 and $\hat{\sigma} = 5.0$. You can confirm that PCR = 1.0



- z for LSL $= (65 - 80)/5.0 = -3$
- z for USL $= (95 - 80)/5.0 = +3$

Shaded area probability $= 100 - 99.73 = 0.27\%$ (3 out of every 1000 products)

What is a good process capability ratio?

- Cp $=$ 1.3: minimum requirement
- Cp $=$ 1.7: requested for safety and other critical applications.
- Cp $=$ 2.0: termed a 6-sigma process: it can move 6σ units left or 6σ units right.

A process with Cp $=$ 1.0 is not a 6σ process.

The above also applies for an uncentered process (next).

Interpreting and using the PCR value for an uncentered process

Closer to the upper specification?

$$\mathrm{PCR_k} = \mathrm{C_{pk}} = \frac{\text{Upper specification limit} - \bar{\bar{x}}}{3\sigma}$$

Closer to the lower specification?

$$\mathrm{PCR_k} = \mathrm{C_{pk}} = \frac{\bar{\bar{x}} - \text{Lower specification limit}}{3\sigma}$$

Combined equation

$$\mathrm{PCR_k} = \mathrm{C_{pk}} = \min\left(\frac{\text{Upper specification limit} - \bar{\bar{x}}}{3\sigma}; \frac{\bar{\bar{x}} - \text{Lower specification limit}}{3\sigma}\right)$$

- $\bar{\bar{x}}$ from Shewhart chart, or simply the target value
- Cpk is a one-sided limit: taken on the worst side!
- Cp is for the balanced (centered) case, which is very unrealistic in practice.

The use of the PCR value in practice

- Cp, and more so, Cpk, is regularly used
- Notice, you don't give away any "trade secrets" by quoting it
- Quick, and easy, to calculate (as long as the process is stable)
- Being dimensionless, it is easily interpreted: higher is better

- Purchasers seek out companies that can provide a product produced on a high PCR process
- Companies reward employees for increasing the PCR
- Even if not, it is a great way for you to track yourself, to check if you made an improvement.

There is much more to “6σ” though

PCR is widely used in the area of 6σ, but “”’6σ” is a general term:

Example 1: Adjust the capability ratio of an existing process

The current estimate of the process capability ratio for a key quality variable was 1.30.
The quality variable has an average value of 64.0.
The process operates closer to the lower specification limit of 56.0.
The upper specification limit is 93.0.

What are the two parameters of the system you could adjust, and by how much, to achieve a capability ratio of 1.67 required by recent safety regulations.
Assume you can adjust these parameters independently.

Solution

Adjust this parameter: ________

Current value: ____________

New value: ____________

Adjust this parameter: ________

Current value: ____________

New value: ____________

Example 1: Adjust the capability ratio of an existing process

The current estimate of the process capability ratio for a key quality variable was 1.30.
The quality variable has an average value of 64.0.
The process operates closer to the lower specification limit of 56.0.
The upper specification limit is 93.0.

What are the two parameters of the system you could adjust, and by how much, to achieve a capability ratio of 1.67 required by recent safety regulations.
Assume you can adjust these parameters independently.

Solution

Adjust this parameter: $\overline{\overline{x}}$

Current value: $\overline{\overline{x}} = 64$

New value: $\overline{\overline{x}} = 66.3$

Adjust this parameter: σ

Current value: $\sigma = 2.05$

New value: $\sigma = 1.60$

Example 2: calculating the capability ratio for an existing process

The following values are the particle size of the most recent 20 shipments from a supplier, taken from their certificates of analysis (C of A):

50.9, 52.9, 51.6, 50.8, 54.6, 52.9, 53.1
48.4, 51.6, 53.1, 53.8, 52.4, 53.1, 50.8
54.6, 52.9, 50.0, 53.8, 54.6, 52.2

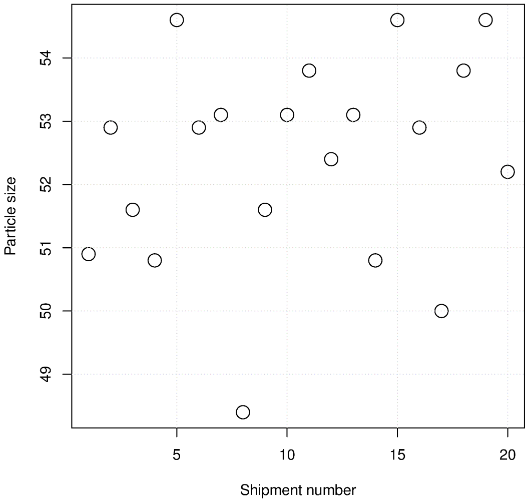
Calculate the supplier's capability, given, from the C of A that:

- the lower specification limit $= 45\mu$m
- the upper specification limit $= 59\mu$m

Clearly state all assumptions you make during the calculations.
To help you, the median of the above data is 52.9μm, and the standard deviation is 1.64μm.

Example 2: calculating the capability ratio for an existing process: solution

We can create this plot from the raw data:



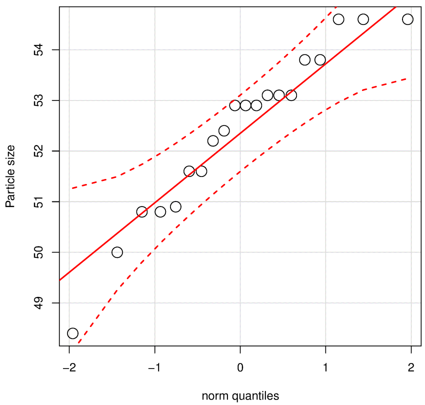
and calculate the following characteristics:

- median $= 52.9\mu$m
- average$= 52.4\mu$m
- standard deviation is 1.64μm
- MAD $= 1.63\mu$m

The above 4 numbers indicate the data have no outliers (why?). The plot simply confirms this visually.

Next, we state the assumptions and perform the calculations.

Example 2: calculating the capability ratio for an existing process: solution



Assumptions made:

1. The prior slides shows there are no outliers. This confirms the assumption the process was under stable operation.
2. The q-q plot shows the process data are normally distributed, which is another assumption required for interpreting the capability ratio. The plot also confirms, again, there are no batches from unstable operation.

Should we calculate C_{pk} (uncentered), or C_p (which assumes a centered process)?

We can verify that the median is not centered, so the assumption of a centered process is not required. Therefore: $C_{pk} = \dfrac{\text{Upper specification limit} - \bar{\bar{x}}}{3\sigma} = \dfrac{59 - 52.9}{3 \times 1.64} = 1.24$

Example 3: understanding the relationship between control and specification limits

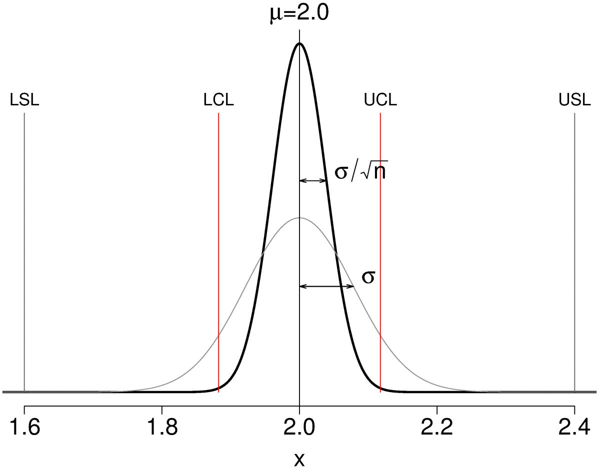
Plastic sheets are manufactured on your blown film line. The Cp value is 1.7.
You sell the plastic sheets to your customers with specification of 2 mm $\pm$ 0.4 mm.

1. List three important assumptions you must make to interpret the Cp value.
2. What is the theoretical process standard deviation, σ?
3. What would be the Shewhart chart limits for this system using subgroups of size $n = 4$?

Illustrate your answer from part 2 and 3 of this question on a diagram of the normal distribution.

Example 3: understanding the relationship between control and specification limits: solution

1. Assumptions required:
   1.1 the process values follow a normal distribution,
   1.2 the process was centered when the data were collected,
   1.3 that the process was stable.
2. The range from the lower to the upper specification limit is 0.8 mm, which spans 6 standard deviations. Given the Cp value of 1.7, the process standard deviation must have been $\sigma = \frac{0.8}{1.7 \times 6} = 0.0784$ mm.
3. The Shewhart control limits would be: $\overline{\overline{x}} \pm 3 \times \frac{\sigma}{\sqrt{n}} = 2 \pm 3 \times 0.0784/2$. The LCL $=$ 1.88 mm and the UCL $=$ 2.12 mm.



Example 1: Adjust the capability ratio of an existing process

The current estimate of the process capability ratio for a key quality variable was 1.30.
The quality variable has an average value of 64.0.
The process operates closer to the lower specification limit of 56.0.
The upper specification limit is 93.0.

What are the two parameters of the system you could adjust, and by how much, to achieve a capability ratio of 1.67 required by recent safety regulations.
Assume you can adjust these parameters independently.

Solution

Adjust this parameter: ________

Current value: ____________

New value: ____________

Adjust this parameter: ________

Current value: ____________

New value: ____________

Example 1: Adjust the capability ratio of an existing process

The current estimate of the process capability ratio for a key quality variable was 1.30.
The quality variable has an average value of 64.0.
The process operates closer to the lower specification limit of 56.0.
The upper specification limit is 93.0.

What are the two parameters of the system you could adjust, and by how much, to achieve a capability ratio of 1.67 required by recent safety regulations.
Assume you can adjust these parameters independently.

Solution

Adjust this parameter: $\overline{\overline{x}}$

Current value: $\overline{\overline{x}} = 64$

New value: $\overline{\overline{x}} = 66.3$

Adjust this parameter: σ

Current value: $\sigma = 2.05$

New value: $\sigma = 1.60$

Example 2: calculating the capability ratio for an existing process

The following values are the particle size of the most recent 20 shipments from a supplier, taken from their certificates of analysis (C of A):

50.9, 52.9, 51.6, 50.8, 54.6, 52.9, 53.1
48.4, 51.6, 53.1, 53.8, 52.4, 53.1, 50.8
54.6, 52.9, 50.0, 53.8, 54.6, 52.2

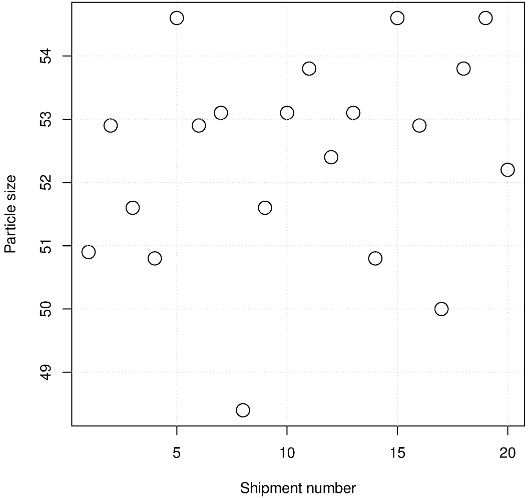
Calculate the supplier's capability, given, from the C of A that:

- the lower specification limit $= 45\mu$m
- the upper specification limit $= 59\mu$m

Clearly state all assumptions you make during the calculations.
To help you, the median of the above data is 52.9μm, and the standard deviation is 1.64μm.

Example 2: calculating the capability ratio for an existing process: solution

We can create this plot from the raw data:



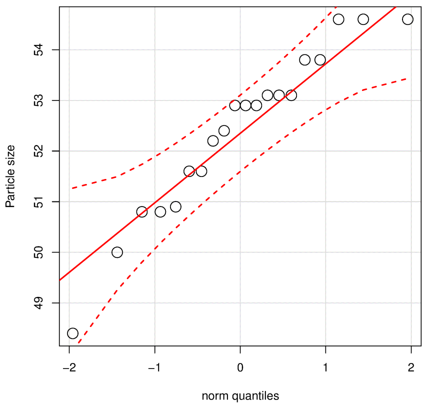
and calculate the following characteristics:

- median $= 52.9\mu$m
- average$= 52.4\mu$m
- standard deviation is 1.64μm
- MAD $= 1.63\mu$m

The above 4 numbers indicate the data have no outliers (why?). The plot simply confirms this visually.

Next, we state the assumptions and perform the calculations.

Example 2: calculating the capability ratio for an existing process: solution



Assumptions made:

1. The prior slides shows there are no outliers. This confirms the assumption the process was under stable operation.
2. The q-q plot shows the process data are normally distributed, which is another assumption required for interpreting the capability ratio. The plot also confirms, again, there are no batches from unstable operation.

Should we calculate C_{pk} (uncentered), or C_p (which assumes a centered process)?

We can verify that the median is not centered, so the assumption of a centered process is not required. Therefore: $C_{pk} = \dfrac{\text{Upper specification limit} - \bar{\bar{x}}}{3\sigma} = \dfrac{59 - 52.9}{3 \times 1.64} = 1.24$

Example 3: understanding the relationship between control and specification limits

Plastic sheets are manufactured on your blown film line. The Cp value is 1.7.
You sell the plastic sheets to your customers with specification of 2 mm $\pm$ 0.4 mm.

1. List three important assumptions you must make to interpret the Cp value.
2. What is the theoretical process standard deviation, σ?
3. What would be the Shewhart chart limits for this system using subgroups of size $n = 4$?

Illustrate your answer from part 2 and 3 of this question on a diagram of the normal distribution.

Example 3: understanding the relationship between control and specification limits: solution

1. Assumptions required:
   1.1 the process values follow a normal distribution,
   1.2 the process was centered when the data were collected,
   1.3 that the process was stable.
2. The range from the lower to the upper specification limit is 0.8 mm, which spans 6 standard deviations. Given the Cp value of 1.7, the process standard deviation must have been $\sigma = \frac{0.8}{1.7 \times 6} = 0.0784$ mm.
3. The Shewhart control limits would be: $\overline{\overline{x}} \pm 3 \times \frac{\sigma}{\sqrt{n}} = 2 \pm 3 \times 0.0784/2$. The LCL $=$ 1.88 mm and the UCL $=$ 2.12 mm.