Statistics for Engineering, 4C3/6C3, 2012

Assignment 7

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Question 1 [0]

Please try question 2 from the course notes in the DOE section: "*Your family runs a small business selling low dollar value products over the web* ...". The solution is provided in the notes, but please attempt the question without looking at the solution. This example will help you interpret interaction effects.

Question 2 [14 = 2 + 1 + 4 + 7]

From the 2011 final exam. One of the experiment projects investigated by a previous student of this course was understanding effects related to the preparation of uncooked, breaded chicken strips. The student investigated 3 factors in a full factorial design (the student actually investigated 4 factors, but found the effect of oven temperature to be negligible!).

- **D** = duration: low level at 15 minutes; and high level = 22 minutes.
- **R** = position of oven rack: low level = use middle rack; high level = use low oven rack (this coding, *though unusual*, was used because the lower rack applies more heat to the food).
- **P** = preheated oven or not: low level = short preheat (30 seconds); high level = complete preheating.

The response variable was y = taste, the average of several tasters, with higher values being more desirable.

Experiment	D	R	P	Taste
1	$-$	$-$	$-$	3
2	$+$	$-$	$-$	9
3	$-$	$+$	$-$	3
4	$+$	$+$	$-$	7
5	$-$	$-$	$+$	3
6	$+$	$-$	$+$	10
7	$-$	$+$	$+$	4
8	$+$	$+$	$+$	7

A full factorial model, using the usual coding, was calculated from these 8 experiments:

$$y = 5.75 + 2.5x_{\text{D}} - 0.5x_{\text{R}} + 0.25x_{\text{P}} - 0.75x_{\text{D}}x_{\text{R}} - 0.0x_{\text{D}}x_{\text{P}} - 0.0x_{\text{R}}x_{\text{P}} - 0.25x_{\text{D}}x_{\text{R}}x_{\text{P}}$$

1. What is the physical interpretation of the $+2.5x_{\text{D}}$ term in the model?
2. From the above table, at what real-world conditions should you run the system to get the highest taste level?
3. Does your previous answer match the above model equation? Explain, in particular, how the non-zero *two factor* interaction term affects taste, and whether the interaction term reinforces the taste response variable, or counteracts it, when the settings you identified in part 2 are used.
4. If you decided to investigate this system, but only had time to run 4 experiments, write out the fractional factorial table that would use factors **D** and **R** as your main effects and confound factor **P** on the **DR** interaction.

   Now add to your table the response column for taste, extracting the relevant experiments from the above table.

   Next, write out the model equation and estimate the 4 model parameters from your reduced set of experiments. Compare and comment on your model coefficients, relative to the full model equation from all 8 experiments.

Question 3 [8]

Problem 8.4 from A First Course in Design and Analysis of Experiments by Oehlert, a free textbook on designed experiments.

Implantable heart pacemakers contain small circuit boards called substrates. These substrates are assembled, cut to shape, and fired. Some of the substrates will separate, or delaminate, making them useless. The purpose of this experiment was to study the effects of three factors on the rate of delamination. The factors were

- **A**: firing profile time, 8 ($-$) versus 13 ($+$) hours with the theory suggesting 13 hours is better;
- **B**: furnace airflow, low ($-$) versus high ($+$), with theory suggesting high is better; and
- **C**: laser, old ($-$) versus new ($+$), with theory suggesting new cutting lasers are better.

A large number of raw, assembled substrates are divided into sixteen groups. These sixteen groups are assigned at random to the eight factor-level combinations of the three factors, two groups (i.e. replicates) per combination. The substrates are then processed, and the response is the fraction of substrates that delaminate. Data from Todd Kerkow.

1. Analyze these data (by hand) to determine how the treatments affect delamination. **Interpret your results**.
2. Verify your calculations using computer software.

Experiment	A	B	C	Delamination fraction
1	$-$	$-$	$-$	0.83 and 0.78
2	$+$	$-$	$-$	0.18 and 0.16
3	$-$	$+$	$-$	0.68 and 0.90
4	$+$	$+$	$-$	0.25 and 0.20
5	$-$	$-$	$+$	0.86 and 0.67
6	$+$	$-$	$+$	0.30 and 0.23
7	$-$	$+$	$+$	0.72 and 0.81
8	$+$	$+$	$+$	0.10 and 0.14

Question 4 [18]

This is a tutorial-type question: all the sub-questions build on each other. All questions deal with a hypothetical bioreactor system, and we are investigating four factors:

- **A** = feed rate: slow or medium
- **B** = initial inoculant size (300g or 700g)
- **C** = feed substrate concentration (40 g/L or 60 g/L)
- **D** = dissolved oxygen set-point (4mg/L or 6 mg/L)

The 16 experiments from a full factorial, 2^4, were randomly run, and the yields from the bioreactor, y, are reported here in standard order: y = [60, 59, 63, 61, 69, 61, 94, 93, 56, 63, 70, 65, 44, 45, 78, 77].

1. Calculate the 15 main effects and interactions and the intercept, using computer software.
2. Use a Pareto-plot to identify the significant effects. What would be your advice to your colleagues to improve the yield?
3. Refit the model using only the significant terms identified in the second question.
   - Clearly explain why you don't actually have to recalculate the least squares model parameters.
   - Compute the standard error and confirm that the effects are indeed significant at the 95% level.
4. Write down the exact settings for **A**, **B**, **C**, and **D** you would provide to the graduate student running a half-fraction in 8 runs for this system.

5. Before the half-fraction experiments are run you can calculate which variables will be confounded (aliased) with each other. Report the confounding pattern for these main effects and for these two-factor interactions. Your answer should be in this format:

   - Generator =
   - Defining relationship =
   - Confounding pattern:
     - $\widehat{\beta}_{\mathbf{A}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{B}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{C}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{D}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{AB}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{AC}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{AD}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{BC}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{BD}} \rightarrow$
     - $\widehat{\beta}_{\mathbf{CD}} \rightarrow$

6. Now use the 8 yield values corresponding to your half fraction, and calculate as many parameters (intercept, main effects, interactions) as you can.

   - Report their numeric values.
   - Compare your parameters from this half-fraction (8 runs) to those from the full factorial (16 runs). Was much lost by running the half fraction?
   - What was the resolution of the half-fraction?
   - What is the projectivity of this half-fraction? And what does this mean in light of the fact that factor **A** was shown to be unimportant?

END