Separation Processes, ChE 4M3, 2012

Assignment 3

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Objectives: Solid-fluid separation systems: particle size characterization, centrifuges, and cyclones.

Question 1 [6]

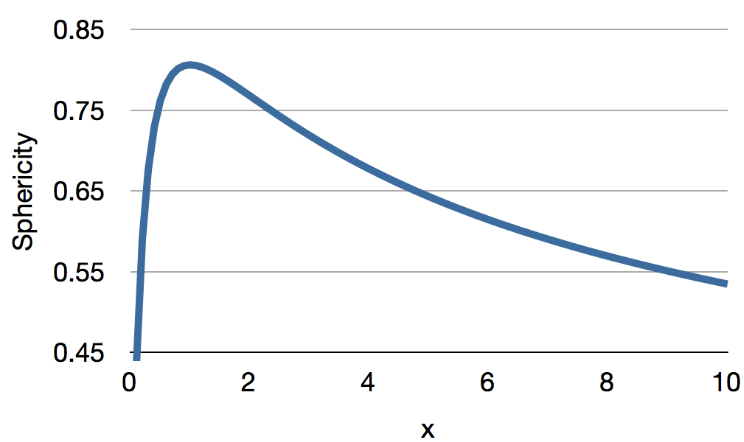
Calculate the sphericity of a rectangular object whose sides are in the ratio $1 : 1 : x$, where x is any value between 1 and 10. Plot the result as x vs sphericity (y-axis).

Solution

$$\text{Sphericity} = \psi = \frac{\text{surface area of sphere with same volume as particle}}{\text{surface area of particle}}$$

- Volume of particle = $V = x$
- General equation for volume of a sphere = $\frac{4}{3}\pi r^3$
- General equation for the surface area of a sphere = $4\pi r^2$
- A sphere with a volume of $V = x$ will have radius = $\sqrt[3]{\frac{3x}{4\pi}}$
- So surface area of sphere with this radius: $4\pi\left(\sqrt[3]{\frac{3x}{4\pi}}\right)^2$
- Surface area of particle = $1 + 1 + x + x + x + x$
- $\psi = \dfrac{4\pi\left(\sqrt[3]{\frac{3x}{4\pi}}\right)^2}{2 + 4x}$

A plot of sphericity with $0.1 \leq x \leq 10$ shows the sphericity increasing to a maximum when $x = 1$, then declining after that again, which makes intuitive sense. It was only required to plot $1.0 \leq x \leq 10$, but additional range is informative.



Question 2 [10]

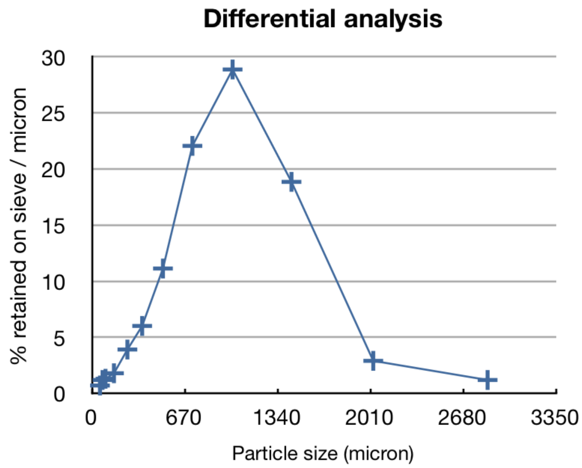
Plot the differential and cumulative analysis (only show the percent passing curve) for the following sieve test results:

Mesh number	Mass retained [g]
6	0.0
8	8.8
12	21.3
16	138.2
20	211.6
30	161.7
40	81.6
50	44.1
70	28.7
140	13.2
170	9.6
230	8.8
Pan	5.2

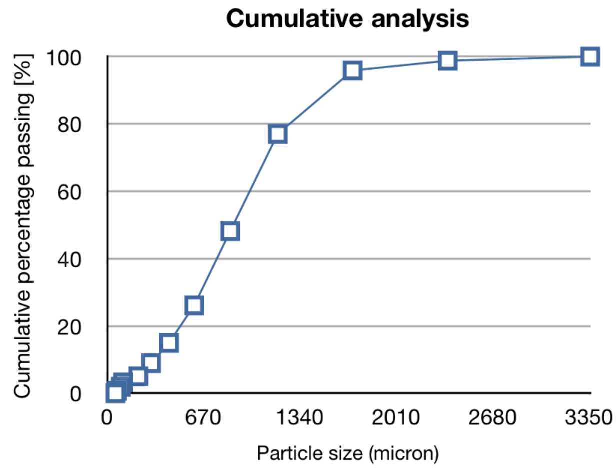
Solution

Mesh	Aperture	Avg aperture	Mass retained	% retained	Cuml % passing
[-]	[micron]	[micron]	[g]	[%]	[%]
6	3350		0	0.0	100
8	2360	2855	8.8	1.2	98.8
12	1700	2030	21.3	2.9	95.9
16	1180	1440	138.2	18.9	77.0
20	850	1015	211.6	28.9	48.2
30	600	725	161.7	22.1	26.1
40	425	512.5	81.6	11.1	15.0
50	300	362.5	44.1	6.0	8.9
70	212	256	28.7	3.9	5.0
140	106	159	13.2	1.8	3.2
170	90	98	9.6	1.3	1.9
230	63	76.5	8.8	1.2	0.7
Pan	~ 53	58	5.2	0.7	0.0

The plots of these data are shown below:



Plot using average screen sizes



Plot using actual screen sizes

Question 3 [4]

Explain why it is possible to operate most cyclones upside down, or rotated at any angle for that matter. Also explain why some centrifuges are mounted so they spin along a horizontal rather than vertical axis.

Solution

Both cyclones and centrifuges use the centrifugal force developed by the angular velocity, ω to achieve the separation. The number of times that this force exceeds gravity, g is given by $\dfrac{r\omega^2}{g}$.

In both situations above the numerator term is so large compared to g that we can essentially ignore gravity effects. The practical implication of this is that we can operate these units in any orientation.

In centrifuges, a higher radius places a greater stress on the material of the outer wall (this is the force that is equal and opposite to the centrifugal force). So narrow diameter centrifuges are preferred, but to obtain high throughput, we would compensate by designing a longer length centrifuge. It is easier to engineer these on a horizontal, rather than vertical axis, for stability reasons.

Question 4 [20]

In class we considered a very small laboratory tubular bowl centrifuge used to separate bacteria from a fermentation broth:

- $r_1 = 16.5$ mm
- $r_2 = 22.2$ mm
- bowl height of 115 mm
- operated at 800 revolutions per second

We calculated the two different throughput flow rates, Q, when integrating between:

- height 0 and h and the particle travelling from $r = r_1$ to $r = r_2$ in this time; called Q_{max}
- height 0 and h and the particle travelling from $r = r_1$ to $r = \dfrac{r_1 + r_2}{2}$; called Q_{cut}

1. Calculate the Σ value for this laboratory centrifuge.
2. We are looking at scaling up from the lab to a larger scale. If we have an existing pilot-plant scale tubular bowl centrifuge, what volume of the same broth can we separate per day? The pilot plant centrifuge has a height of 30 cm, an inner weir radius of 5 cm and an outer radius of 10 cm. It can be operated between 0 and 20,000 revolutions per minute. Recall the particles in the broth had an estimated density of 1040kg.m^{-3} and a diameter of 0.7 μm. The broth had a density of 1010kg.m^{-3} and viscosity of 0.001 kg.m$^{-1}.s^{-1}$.
3. In reality the bacteria are not of uniform size. Plot operating curves that shows the smallest particle diameter we can remove from the broth on the pilot scale unit. Show the result as a function of the throughput flow rate Q_{cut}, with operating curves at 5000, 10000, 15000 and 20000 rpm. These plots will help the engineering team responsible for scale-up select a flow rate and operating speed to remove the smallest diameter particle required with minimum energy cost.

Solution

1. The Σ value for a tubular bowl centrifuge is

$$\begin{aligned}\Sigma &= \frac{\omega^2\left[\pi h\left(r_2^2 - r_1^2\right)\right]}{2g\ln\left[2r_2/(r_1+r_2)\right]}\\ &= \frac{(5026)^2\left[\pi(0.115)\left((0.0222)^2-(0.0165)^2\right)\right]}{2(9.81)\ln\left[2\times 0.0222/(0.0165+0.0222)\right]}\\ &= 747\text{m}^2\end{aligned}$$

2. The larger scale centrifuge, when operated at its highest rotational speed has a Σ value of

$$\begin{aligned}\Sigma &= \frac{(2094)^2\left[\pi(0.3)\left((0.1)^2-(0.05)^2\right)\right]}{2(9.81)\ln\left[2\times 0.1/(0.05+0.1)\right]}\\ &= 5493\text{m}^2\end{aligned}$$

so it the equivalent separating area that is about 7.3 times greater than the lab centrifuge.

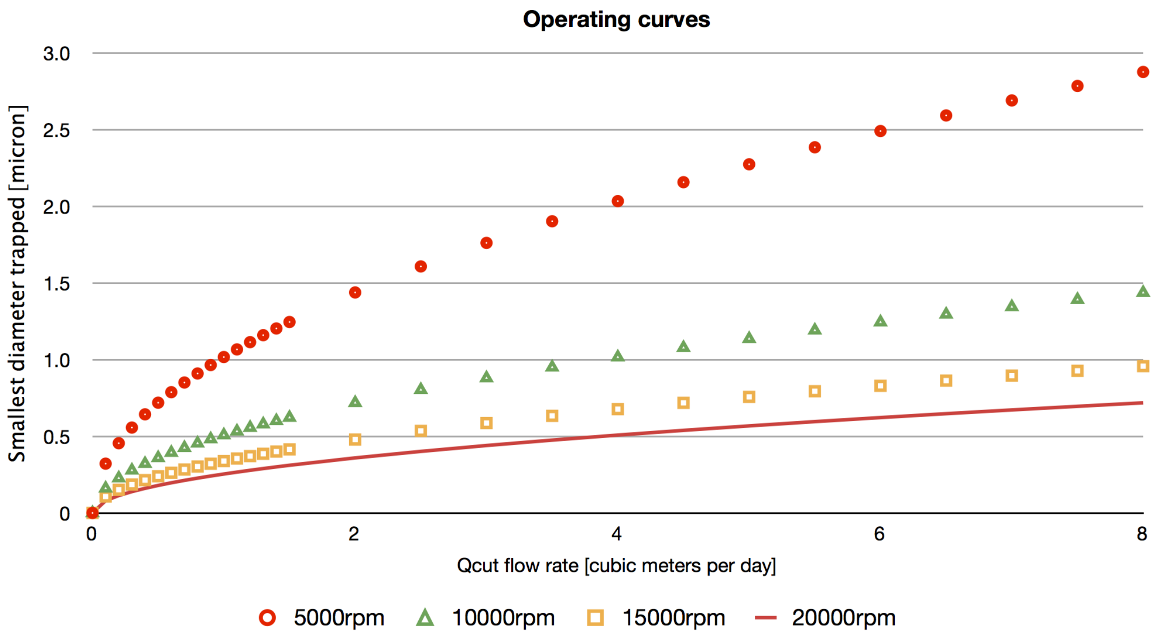
The greatest volume of broth that can be separated, based on Q_{cut} is

$$\begin{aligned}\frac{Q_{\text{cut, pp}}}{\Sigma_{\text{pp}}} &= \frac{Q_{\text{cut, lab}}}{\Sigma_{\text{lab}}}\\ Q_{\text{cut, pp}} &= \frac{Q_{\text{cut, lab}}}{747}\cdot 5493 = 7.3\,Q_{\text{cut, lab}}\\ Q_{\text{cut,lab}} &= \frac{D_{p,\text{cut}}^2\left(\rho_p-\rho_f\right)\omega^2}{18\mu_f\ln\left[2r_2/(r_1+r_2)\right]}\pi\left(r_2^2-r_1^2\right)h\\ &= \frac{(0.7\times 10^{-6})^2\,(1040-1010)\,(2094)^2}{18(0.001)\ln\left[2\times 0.1/(0.05+0.1)\right]}\pi\left((0.1)^2-(0.05)^2\right)(0.3)\\ Q_{\text{cut, pp}} &= (7.3)(1.197\times 10^{-5})\text{ m}^3.\text{s}^{-1} = \mathbf{8.7\times 10^{-5}\ m^3.s^{-1}}\\ &\equiv 7.6\text{ m}^3.\text{day}^{-1}\end{aligned}$$

3. We aim to have operating curves over a reasonable range. From the previous question, at maximum ω was can achieve our highest throughput, Q_{cut} for 0.7 μm particles. This gives an idea of the range over which we can operate. The curves below could be extended up to about 20 m^3.day^{-1}, after which they behave fairly linearly.

   To plot the operating curves we will select flow rates Q_{cut} and solve the above equation for $D_{p,\text{cut}}$ for a given value of $\omega = [5000, 10000, 15000, 20000]$ rpm $\equiv [523, 1047, 1571, 2094]$ rad.s^{-1}.

$$D^2_{p,\text{cut}} = \frac{Q_{\text{cut,lab}} 18 \mu_f \ln\left[2r_2/(r_1+r_2)\right]}{(\rho_p - \rho_f)\,\omega^2 \pi \left(r_2^2 - r_1^2\right) h}$$



   Notice that the engineers can now trade off energy consumption from ω against throughput for a given particle size. e.g. to capture 1 μm particles we can operate the unit at 5000, 10000 or 15000 rpm. At higher energy use we can achieve higher throughputs. Conversely, for a given throughput, we can select a different operating speed and isolate smaller and smaller particles at higher speeds.

Question 5 [20]

A cyclone is being fed with a dust-laden stream at a rate of 200 kg solids per hour, with most of the solids leaving in the underflow at about 130 kg solids per hour. These streams were sampled for a period of time and using screens, the following size analyses were performed:

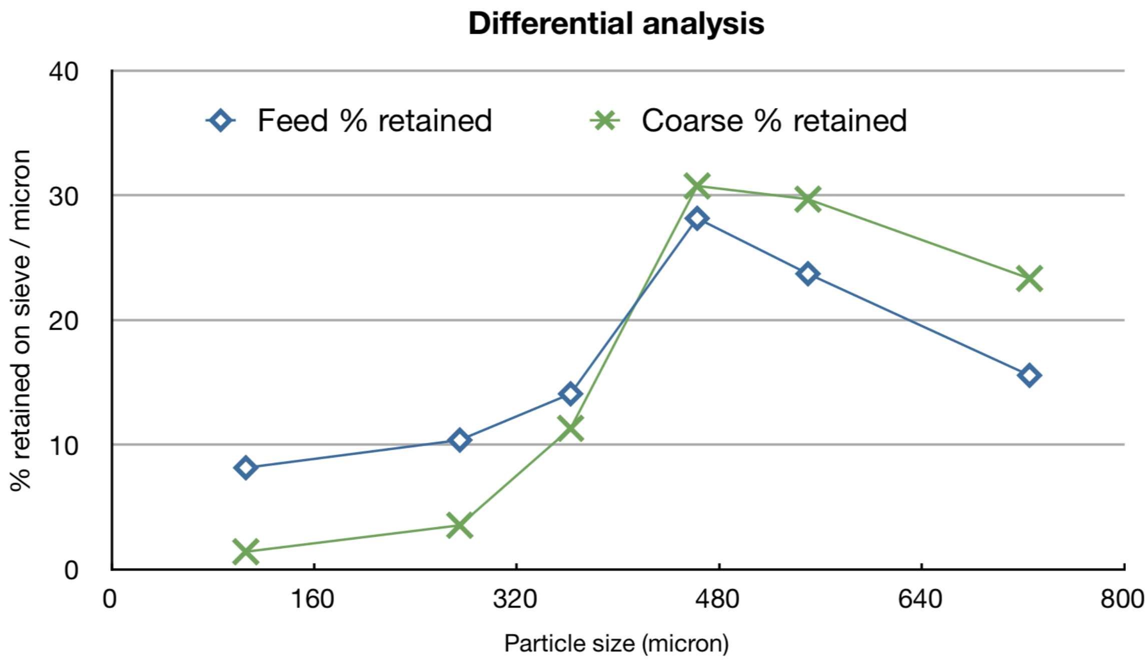
Mesh number	Mass retained from feed [g]	Mass retained from coarse stream [g]
20	0	0
30	21	66
35	32	84
40	38	87
50	19	32
60	14	10
Pan	11	4

1. Plot the differential analysis curve for both streams, superimposed.

2. Calculate the cut size.
3. Describe one way you could **quantify** the cut's sharpness.

Solution

1. A differential analysis of the feed and coarse stream are shown below, with the numeric values shown in part 2.



2. The cut size is calculated from the grade efficiency curve:

Mesh	Aperture	Avg aperture	Feed frac	Coarse frac	Feed flow	Coarse flow	$G(x)$
[-]	[micron]	[micron]	[%]	[%]	[kg/hr]	[kg/hr]	[fraction]
20	850	850	0.0	0.0	0.0	0.0	1.000
30	600	725	15.6	23.3	31.1	30.3	0.975
35	500	550	23.7	29.7	47.4	38.6	0.814
40	425	463	28.1	30.7	56.3	40.0	0.710
50	300	362	14.1	11.3	28.1	14.7	0.522
60	250	275	10.4	3.5	20.7	4.6	0.221
Pan	~ 212	231	8.1	1.4	16.3	1.8	0.113

The cut size is the size fraction that is at $G(x) = 0.5$, which is around 350 μm in this case. **Note**: the grade efficiency curve is sometimes calculated around the actual sieve size, rather than the averages, so either an answer of 350 or 300 μm would be acceptable.

3. One way is to estimate the slope around the $G(x) = 0.5$ point: the steeper the slope, the greater the sharpness. Some people use the ratio of 80% to 20% cuts, others the ratio of the 75% to 25% cuts. This ratio can be compared on different cyclones.

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