Separation Processes, ChE 4M3, 2014

Assignment 4

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Objectives: This assignment is to gain some experience with the membrane calculations.

Question 1 [28 = 5 + 2 + 12 + 2 + 3 + 2 + 2]

A reverse osmosis plant treats $120,000\,\text{m}^3$ of seawater per 8 hours, at 20°C and 3.5 wt% solids (assume it to be NaCl). The molar mass of NaCl is 58.4 g/mol and is 18.02 g/mol for water. The aim is to obtain $35,000\,\text{m}^3$ of drinking water within an 8 hour period, with only 500 ppm (0.05 wt%) dissolved solids in it.

The feed pressure is 140 atm entering and leaving at 4 atm in the permeate. The total area of the spiral wound membranes is $180,000\,\text{m}^2$. The plant only operates 8 hours per day, in the evenings, when electricity is cheapest. Storage tanks are used to hold the water produced during the 8 hours, so that it is available 24 hours per day to the town.

From lab experiments at the supplier, the permeance of water through a single membrane module was found to be $5.5 \times 10^{-5}\,\text{kg.s}^{-1}.\text{m}^{-2}.\text{atm}^{-1}$. The permeance of salt through the membrane was $21 \times 10^{-8}\,\text{m.s}^{-1}$.

1. Give a few bullet points that describe how the membrane's permeance with respect to water is calculated. Your description must take the given units into account. **[5]**
2. Is that water permeance value applicable to all $180,000\,\text{m}^2$ of membrane area? Explain. **[2]**
3. Calculate the actual flow rate of drinking water leaving the plant. **[12]**
4. Will the drinking water flow meet the demand required? If the demand cannot be met, name one thing that can be improved or changed to meet demand. **[2]**
5. Is this flux close to typical LMH values experienced on reverse osmosis applications? Explain why. **[3]**
6. What is the rejection coefficient for this system? **[2]**
7. What is the cut value? **[2]**

Solution

This question was from the final exam in 2013.

1. Permeance is the term A_{solv} in the equation $J_v = A_{\text{solv}}\left(\Delta P - \Delta \pi\right)$.

   Using pure water, which has no osmotic pressure, causes $\Delta\pi = 0$.

   Apply a constant pressure difference, ΔP and measure the volume of permeate acquired in a given amount of time for the given membrane area. This gives us $J_v = \dfrac{Q_P}{A}$.

   Solve the equation for $A_{\text{solv}} = \dfrac{J_v}{\Delta P}$ which will have units volume. Using the density of water we can get a mass flux, $J = J_v\rho$, and then use J instead of J_v to calculate the permeance. This will have units, which are the same as in the problem given here.

2. Yes, in general the (global) permeance value calculated is for the entire membrane, determined from experiments, as described above. The exact permeance will vary at different points on the membrane, due to manufacturing imperfections. It should remain constant over time, as long as there is no cake build-up or fouling.

3. Only the final values are given; ensure your answer states assumptions and shows the calculations.

   - $\Delta P = 140 - 4 = 136\,\text{atm}$
   - molar mass of salt = $58.4\,\text{g.mol}^{-1}$
   - $C_{\text{feed}} = \dfrac{3.5\,\text{g salt}}{100 - 3.5\,\text{g water}} \equiv 1242\,\dfrac{\text{mols salt ions}}{\text{m}^3}$
   - $C_{\text{perm}} = \dfrac{500\,\text{g salt}}{10^6 - 500\,\text{g water}} \equiv 17.1\,\dfrac{\text{mols salt ions}}{\text{m}^3}$
   - $\Delta\pi = 29.5\,\text{atm}$
   - $J_v = 5.86 \times 10^{-3}\,\text{kg.s}^{-1}.\text{m}^{-2}$
   - $Q_p = 3796\,\text{m}^3.\text{hr}^{-1}$

4. In 8 hours this corresponds to producing $30,365\,\text{m}^3$ of water instead of the required $35,000\,\text{m}^3$. So we won't meet the requirements. We can increase the pressure difference, add more area (if that were a feasible option, which it might not be), or raise the temperature (this is an evolving area of research to improve flux).

5. Based on the table from Perry's Handbook, which shows an LMH of about 25, our flux of 21.1 LMH is reasonable. That table was for similar salt water concentrations.

6. The rejection coefficient is 98.6% (show calculations though).

7. The cut value is 25.3 (show calculations though).

Question 2 [20]

An asymmetric ultrafiltration membrane is used with the aim of separating dyes from a liquid stream and to achieve a more concentrated dye-water mixture. The feed waste stream arrives at a flow rate of 2.2 $\text{m}^3.\text{hour}^{-1}$ with concentration of 1.2 kg.m^{-3}. The membrane's operating characteristic was calculated from various experiments:

$$J_v = 0.04 \ln\left(\frac{15}{C}\right)$$

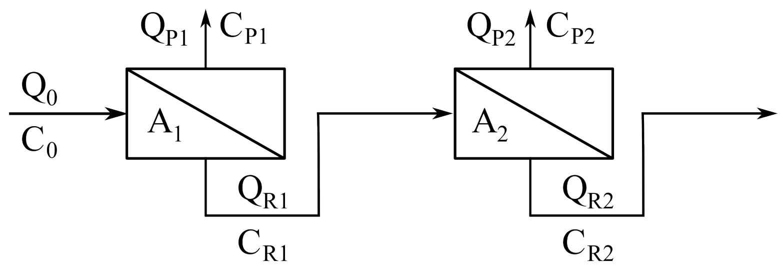
where the bulk concentration C has units of kg.m^{-3} and flux is measured in $\text{m}^3.\text{hour}^{-1}.\text{m}^{-2}$.

If two membrane modules, each of area 25 m^2, are simply placed in series, give reasonable estimates of:

1. the dye concentration from the first membrane module?
2. the permeate flow rate from the first membrane module?
3. the dye concentration from the final membrane module?
4. the permeate flow rate from the final membrane module?
5. Then explain whether the above answers seem reasonable.

Solution

An illustration of the system is



recognizing that it is the retentate from the first module that is fed into the second membrane.

We make the assumption, common in ultrafiltration, that the permeate concentration leaving each module is approximately zero. The flux J_v refers to the flux *through* the membrane, in other words the flux of the permeate. The flux equation is in terms of the bulk concentration in the membrane, which is ultimately the concentration leaving, i.e. C_{R1} and C_{R2} for module 1 and 2 respectively.

We solve the membrane modules in series, because there are no recycle loops or coupling between the modules. We can cascade down from module to module. Mass balance on module 1:

$$Q_0 C_0 = Q_{R1} C_{R1} + Q_{P1} \cancelto{0}{C_{P1}} \tag{1}$$

Volume balance on module 1:

$$Q_0 = Q_{R1} + Q_{P1} \tag{2}$$

Solving for C_{R1}, the first part of the question, using (1) and substituting in (2):

$$C_{R1} = \frac{Q_0 C_0}{Q_{R1}} = \frac{Q_0 C_0}{Q_0 - Q_{P1}} \tag{3}$$

however we can use the flux equation to substitute for Q_{P1}:

$$J_v = \frac{Q_{P1}}{A_1} = 0.04 \ln\left(\frac{15}{C}\right)$$

Leading to

$$C_{R1} = \frac{Q_0 C_0}{Q_0 - 0.04 A_1 \ln\left(\dfrac{15}{C_{R1}}\right)}$$

simplifying and substituting in known values

$$2.2 C_{R1} - C_{R1}(0.04)(25) \ln\left(\frac{15}{C_{R1}}\right) - (2.2)(1.2) = 0 \tag{4}$$

where equation (4) is in the form $f(C_{R1}) = 0$. A lower bound for C_{R1} is 1.2, while an upper bound is 15 kg.m^{-3}.

Using guess-and-check, or a calculator with root-finding capability:

$$
\begin{aligned}
f(C_{R1} = 3) &= -0.84 \\
f(C_{R1} = 8) &= 9.9 \qquad \text{obviously too high a guess} \\
f(C_{R1} = 4) &= 0.87 \qquad \text{still too high} \\
f(C_{R1} = 3.5) &= -0.03 \qquad \text{close enough} \\
\text{So } C_{R1} &= 3.5 \text{ kg.m}^{-3}
\end{aligned}
$$

From which we can solve for the remaining unknowns in module 1: $Q_{R1} = 0.75$ m^3.hour^{-1} and $Q_{P1} = 1.45$ m^3.hour^{-1}. A similar construction leads to: $C_{R2} = 9.4$ kg.m^{-3}, $Q_{R2} = 0.28$ m^3.hour^{-1} and $Q_{P2} = 0.47$ m^3.hour^{-1}.

The dye concentrations are being upgraded, as expected: from 1.2 to 3.5 kg.m^{-3} across module 1 and from 3.5 to 9.4 kg.m^{-3} in module 2. The corresponding flows leaving in the retentate decrease: from 2.2 m^3.hour^{-1} in the feed to 0.75 and then down to 0.28 m^3.hour^{-1} in the last module.

The permeate stream, essentially pure solvent is the balance of the volume: 1.45 + 0.47 = 1.92 m^3.hour^{-1} of pure liquid.

Grading for this question:

- 2 marks to set up the diagram with the correct flow connections, or it must be *very* clear from your calculations what the connections would have been
- 2 marks for the dye mass balance across module 1
- 2 marks for the volume balance across module 1
- 2 marks for correctly using the flux equation (correct flow term and concentration term)
- 2 marks for substituting equations into each other to attempt to solve for C_{R1}
- 2 marks for attempting to solve the non-linear equations by guess-check or calculator
- 2 marks for the solving for the correct values of C_{R1}, Q_{R1}, Q_{P1}
- 2 marks for setting up the equations for solving module 2
- 2 marks for solving for the actual values in module 2
- 2 marks for interpretation in the final part

END