Separation Processes

ChE 4M3

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- **any suggestions to improve the notes**

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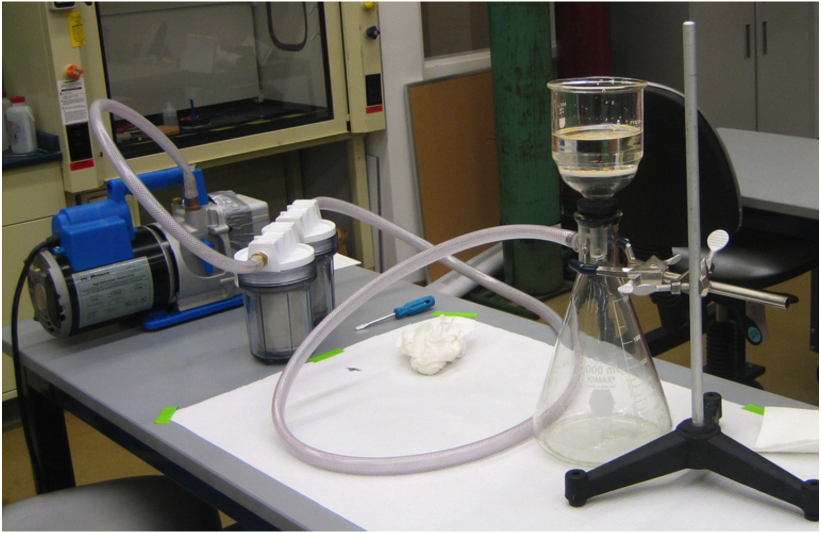
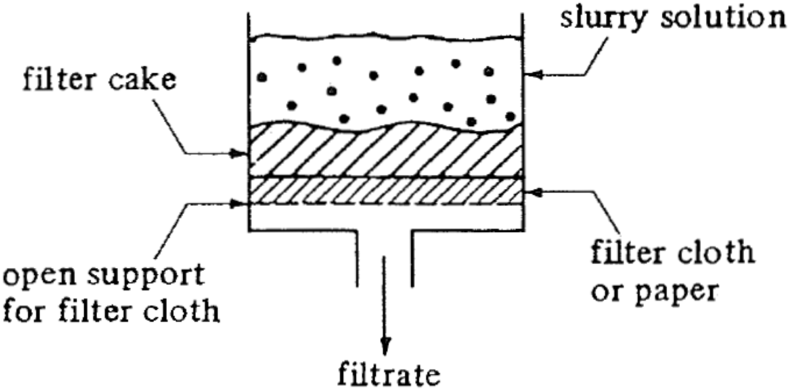
Filtration



Filtration section

Filtration: a pressure difference that causes separation of solids from slurry by means of a porous medium (e.g. filter paper or cloth), which retains the solids and allows the filtrate to pass



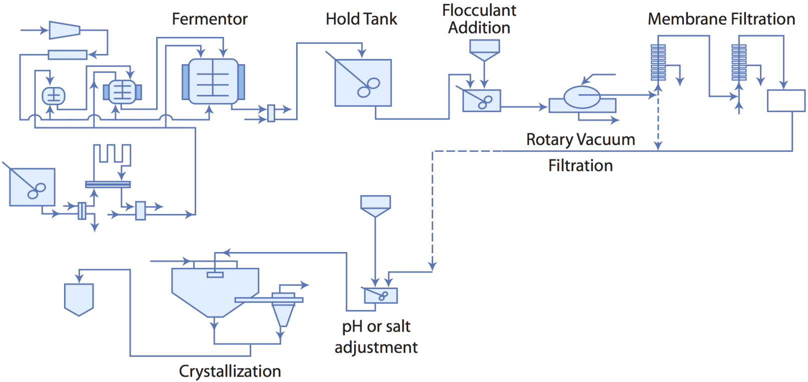
[Geankoplis, p 905]

References on filtration

- Geankoplis, “Transport Processes and Separation Process Principles”, 4th edition, chapter 14.
- Perry’s Chemical Engineers’ Handbook, 8th edition, chapter 18.
- Seader, Henley and Roper, “Separation Process Principles”, 3rd edition, chapter 19.
- Uhlmann’s Encyclopedia, “Filtration 1. Fundamentals”, DOI:10.1002/14356007.b02_10.pub3

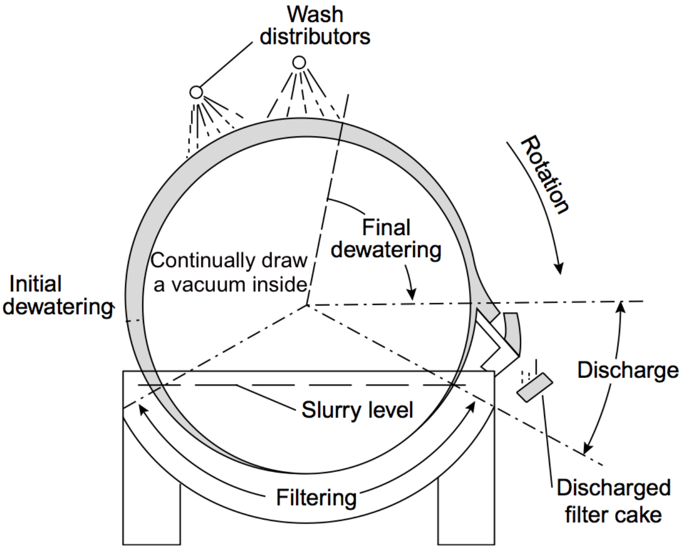
Why filtration?

Example: alkaline protease, used as an additive in laundry detergent



[MIT OCW, Course 10-445, Separation Processes for Biochemical Products, 2005]

Commercial units: rotary drum filter



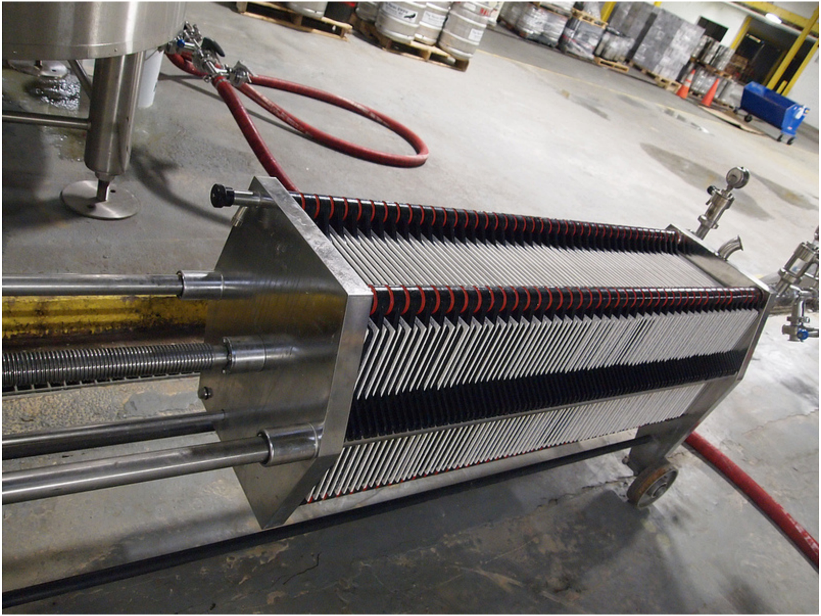
[Seader, modified from fig 19-13] [YouTube video]

Commercial units: plate and frame

Commercial units: plate and frame

[Also see YouTube video]

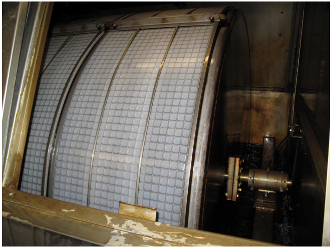
Commercial units: plate and frame (beer clarification)



Questions to discuss

1. What characteristics of a filtration system will you use to **judge** the unit's performance?

2. What factors can be used to **adjust** the units's performance?



Example: a rotary drum filter

Poiseuille's law

Recall from your fluid flow course that **laminar flow** in a pipe (considering no resistance):

$$\frac{-\Delta P}{L_c} = \frac{32\ \mu\ v}{D^2}$$

$-\Delta P$	=	pressure drop from start (high P) to end of tube	[Pa]
L_c	=	length being considered	[m]
μ	=	fluid viscosity	[Pa.s]
v	=	fluid's velocity in the pipe	[m.s^{-1}]
D	=	pipe diameter	[m]

Carmen-Kozeny equation through a bed of solids (cake)

$\dfrac{-\Delta P}{L_c} = \dfrac{32\ \mu\ v}{D^2}$ from which we derive the Carmen-Kozeny equation:

$$\frac{-\Delta P_c}{L_c} = k_1 \cdot \mu \cdot \frac{v}{\epsilon} \cdot \left(\frac{(1-\epsilon)S_0}{\epsilon} \right)^2$$

$-\Delta P_c$	$=$	pressure drop through the cake	[Pa]
k_1	$=$	$4.17,$ a constant	$[-]$
ϵ	$=$	void fraction, or porosity typical values?	$[-]$
S_0	$=$	specific area per unit volume	$[\mathrm{m}^2.\mathrm{m}^{-3} = \mathrm{m}^{-1}]$

- $S_0 =$ specific surface area per unit volume is a property of the solids
- Prove in the next assignment, for spheres, $S_0 = \dfrac{6}{d} = f(d)$

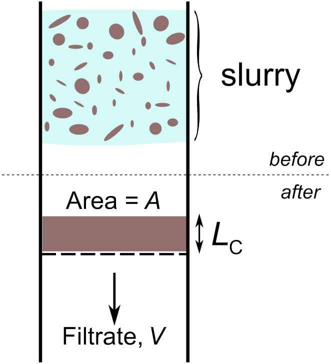
Solids balance

Mass of solids in the filter cake $=$
$A\ L_c\,(1-\epsilon)\,\rho_p$

Mass of fluid trapped in the filter cake $=$
$A\ L_c\ \epsilon\ \rho_f \approx$ small

Mass of fluid in the filtrate $=\ V\rho_f$

What key assumptions are being made here?



ρ_p	$=$	solid particle density	[kg.m^{-3}]
ρ_f	$=$	fluid density	[kg.m^{-3}]
V	$=$	volume of filtrate collected	[m^3]
A	$=$	cross sectional area for filtration	[m^2]

Then define slurry concentration:

$$C_S = \frac{\text{mass of dry solids}}{\text{volume of filtrate}} = \frac{AL_c\,(1-\epsilon)\,\rho_p}{V}$$

Exercise

1. Calculate the mass of solids in the cake for the case when
    - $\rho_p = 3000\text{kg.m}^{-3}$
    - $A = 8\text{m}^2$
    - $L_c = 10\text{cm}$
2. Also calculate the mass of water in the cake.

Deriving the flow through the filter

Our standard equation for fluid flow:

$$\frac{1}{A} \cdot \frac{dV}{dt} = v = \frac{Q}{A}$$

for a given velocity v, and volumetric feed flow rate, Q.

But, from the Carmen-Kozeny equation:

$$\begin{aligned} \frac{-\Delta P_c}{L_c} &= \frac{k_1 \mu v \left(1-\epsilon\right)^2 S_0^2}{\epsilon^3} \\ v &= \frac{\left(-\Delta P_c\right)\left(\epsilon\right)^3}{(\mu)(k_1)(L_c)(1-\epsilon)^2(S_0^2)} \end{aligned}$$

from our definition for C_S we can solve for L_c

$$\begin{aligned} L_c &= \frac{C_S V}{A\left(1-\epsilon\right)\rho_p} \\ \frac{1}{A} \cdot \frac{dV}{dt} = v &= \frac{\left(-\Delta P_c\right)(A)\cancel{(1-\epsilon)}(\epsilon)^3(\rho_p)}{(\mu)(C_S)(V)\cancel{(1-\epsilon)}(k_1)\left(1-\epsilon\right)\left(S_0^2\right)} = \frac{-\Delta P_c}{\mu C_S V \; \alpha \; /A} \end{aligned}$$

The general filtration equation

$$\frac{1}{A}\cdot\frac{dV}{dt} = \frac{-\Delta P_c}{\mu C_S V \alpha / A}$$

$$\begin{aligned} C_S &= \text{slurry concentration} \quad [(\text{kg dry solids})/(\text{m}^3 \text{ filtrate})] \\ \alpha &= \text{specific cake resistance} \quad [\text{m.kg}^{-1}] \end{aligned}$$

All aspects of engineering obey this general law:

$$J = \text{flux} = \frac{\text{transfer rate}}{\text{transfer area}} = \frac{\text{driving force}}{\text{resistance}}$$

including the filtration equation:

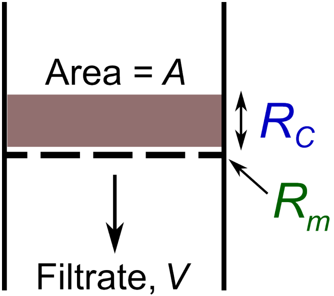
$$\begin{aligned} \frac{1}{A}\cdot\frac{dV}{dt} &= \frac{-\Delta P_c}{\mu C_S V \alpha / A} \\ \frac{1}{A}\cdot\frac{dV}{dt} &= \frac{-\Delta P_c}{\mu R_c} \end{aligned}$$

$$R_c = \text{resistance due to the cake} = \frac{C_S V \alpha}{A} \quad [\text{m}^{-1}]$$

Resistance due to the filter medium

In a similar way, we can define the filter medium's resistance:

$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_m}{\mu R_m}$$



$-\Delta P_m$	=	pressure drop across the medium	[Pa]
R_m	=	resistance due to the filter medium	[m^{-1}]

Notes:

- From a practical standpoint, R_m is empirical for the given filter
- We wrap up all other minor resistances into R_m also (e.g. pipe flow into/out of filter)
- The flux through the filter cake is exactly the same as through the medium
- After filtration gets started, we very often have $R_m \ll R_c$

Bringing it all together

As with resistances in series (you learned in Physics I), we have:

$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_m}{\mu R_m} + \frac{-\Delta P_c}{\mu R_c} = \frac{-\Delta P_{\text{tot}}}{\mu \left(R_m + R_c \right)}$$

this is called the general filtration equation.

$$\begin{aligned} R_c &= \text{resistance due to the cake} && [\text{m}^{-1}] \\ R_m &= \text{resistance due to the medium} && [\text{m}^{-1}] \\ -\Delta P_{\text{tot}} &= \text{total pressure drop} = -(\Delta P_c + P_m) && [\text{Pa}] \end{aligned}$$

Questions

How would you use this equation?

$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_{\text{tot}}}{\mu \left(R_m + R_c \right)}$$

1. to determine the medium resistance?
2. to determine the cake resistance?
3. to find the utility cost of operating the filter?
4. to predict the flow for a given filter when your boss wants a higher throughput?