Separation Processes, ChE 4M3, 2012

Assignment 2

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Objectives: Wrapping up the sedimentation section; dealing with more open-ended questions and moving on to other solid-fluid separation systems.

Question 1

A rectangular settling basin is used for clarifying a mixed water-biomass stream at a feed rate of 130 m^3 per hour. The basin area is 5000 m^2.

There are biomass particles with approximately the following sizes present: 5μm, 8μm, 12μm and 20μm and these particles have a density of 1100kg.m^{-3}.

1. Which particle sizes will be completely separated out?
2. What will happen with the other particle sizes?

Solution

We have the relationship that $A = \dfrac{Q}{v}$ for the cross-sectional area, volumetric flow rate and terminal velocity, v. This relationship holds no matter what the shape of the sedimentation vessel. Using the information provided we can calculate the design settling rate, $v = \dfrac{Q}{A} = \dfrac{130}{5000}\dfrac{\text{m}}{\text{hr}} = 7.2 \times 10^{-6}\text{m.s}^{-1} = 7.2\mu m.\text{s}^{-1}$.

Particles sizes that settle faster than this rate can be assumed to be completely removed according to the original design. Particles sizes settling slower than this will be partially removed, if we assume this to be an ideal sedimentation vessel.

Assuming Stokes' law (we will verify it afterwards): the theoretical terminal velocity of a particle with diameter D_p in water at average ambient conditions:

$$v = \frac{(\rho_p - \rho_f)\, g D_p^2}{18\mu_f} = \frac{(1100 - 1000)\,(9.81) D_p^2}{(18)(0.001)} = 54500 D_p^2$$

D_p [micron]	v_{TSV} [μm.s^{-1}]	Re
5	1.36	6.8×10^{-6}
8	3.5	2.8×10^{-5}
12	7.85	9.4×10^{-5}
20	21.8	4.4×10^{-4}

Any particles with v $< 7.2 \times 10^{-6}$m.s^{-1} will not be completely removed, depending on the height at which they enter the vessel, i.e. the 5μm and 8μm particles will be partially removed, while the others will be completely removed. The Reynolds number is well below 1.0, so Stokes' law assumption was valid.

Question 2

We are designing a new sedimentation unit. A flocculant test on laboratory samples appears to quadruple the settling time from 0.5mm.s^{-1} to almost 2mm.s^{-1} for a given waste stream. What is the effect on the design's tank diameter, and the approximate effect on the capital cost?

Solution

Since $Q = Av$, and Q is assumed fixed for the given waste stream, then a 4-fold increase in settling rate, v means that the area can be reduced 4 times, i.e. the diameter can be halved. The effect on capital cost, given Perry's capital cost estimate = ax^b, with $b = 1.38$ and x is the diameter in feet; this implies the capital cost will be reduced by the following factor:

$$\frac{\text{Cost before}}{\text{Cost after}} = \frac{ax_{\text{before}}^{1.38}}{ax_{\text{after}}^{1.38}} = \frac{ax_{\text{before}}^{1.38}}{a\left(\frac{x_{\text{before}}}{2}\right)^{1.38}} = \frac{1}{2^{-1.38}} = 2.6$$

In other words, the capital costs are reduced by more than half.

Question 3

A thickener is operating at the designed feed rate of 180m^{3}hr^{-1} but needs to be operated at 225m^{3}hr^{-1} due to increased upstream production. It is the last step before discharging the overflow stream to municipal treatment. Since your company is under investigation from government authorities already, there can be absolutely no risk of discharging additional solids in the overflow.

Clearly explain at least 3 options you can realistically investigate to handle the increased flow; and be as creative as possible. Also, be clear on the expected magnitude of your effect: is it linear or some other function?

Short answer: Answers as discussed in class.

Question 4

It is required to settle dust particles from a moving air stream. One option that could be used is to pipe the incoming dust/air mixture to a large rectangular container with a dissipator at the container entrance. A moving conveyor belt on the bottom of the container will remove any solid particles that settle out of the air. The clean(er) air can be withdrawn from the other side of the container. Your company is trying to do this as cheaply as possible, so they are using a standard shipping container.

The particles have density of 1300kg.m^{-3}. Pick a container size and calculate the minimum theoretical particle size that will drop out. Apply an overdesign factor of 10 to the settling velocity. Present your answer as a plot, showing the minimum particle size that can be removed (y-axis) as a function of the volumetric feed flow rate, Q, in units of m^3.s^{-1}, on the x-axis.

Solution

Recognize in this open ended question that this is nothing more than designing a sedimentation vessel, except we have to pick from a few known sizes. Also recognize that we expect the container height should have no influence (see the lecture notes): we will prove this in the answer here.

The viscosity of air is taken as 1.8×10^{-5} kg.(m.s)$^{-1}$. The density of air can effectively be ignored, it is 3 orders of magnitude lower than the solid particle (your answer will hardly change if you do take it into account). However, we do require it to check the Reynolds number assumption later on, so it was taken as 1.2 kg.m^{-3}.

The time take for the particle to settle from a height H is related to its terminal settling velocity = $t_{\text{vert}} = \dfrac{H}{v_{\text{TSV}}/10} = \dfrac{18(10)\mu_f H}{D_p^2\left(\rho_p - \rho_f\right)g}$, which is includes an over design factor of one order of magnitude.

For a given container of width, W, length L and height H, the residence time in the container is $t_{\text{horiz}} = \dfrac{V}{Q}$, which is the complete time the particle spends travelling from entrance to exit. The vertical time, t_{vert}, must be shorter than t_{horiz}, otherwise we will assume the particle was not able to settle out; so it will be removed in the overflow. Note: we could have used, like we did in the derivation for centrifuges, that a particle should reach at least $0.5H$, but we've already got the over-design factor of 10 on the settling velocity.

So then

$$\begin{aligned} t_{\text{horiz}} &> t_{\text{vert}} \\ \frac{V}{Q} &> \frac{18(10)\mu_f H}{D_p^2\left(\rho_p - \rho_f\right)g} \\ D_p^2 &> \frac{18(10)\mu_f HQ}{HWL\left(\rho_p - \rho_f\right)g} \end{aligned}$$

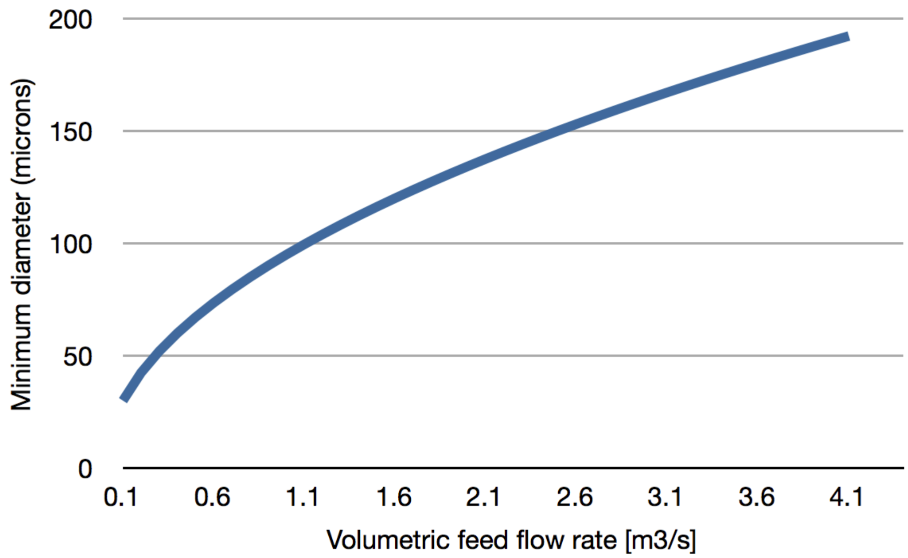
Let's call this diameter $D_{p,\text{min}}$

$$D_{p,\text{min}} > \sqrt{\frac{18(10)\mu_f Q}{WL\left(\rho_p - \rho_f\right)g}} = \sqrt{\frac{18\mu_f(10)}{\left(\rho_p - \rho_f\right)g} \cdot \frac{Q}{A}}$$

Let's make sure this is sensible before substituting in numbers. This equation gives the smallest particle size we can separate out, i.e. any diameter larger than this $D_{p,\text{min}}$ will reach the container's bottom.

- Firstly, we see height cancelling out, which we expect from theory, but great to see it confirmed.
- The over design factor must be in the numerator, showing that the larger the factor, the larger the minimum diameter.
- The physical properties make sense: higher viscosity fluid, smaller density differences of the fluid would increase the minimum diameter, as would increasing the throughput, Q.
- The footprint of the container, $A = WL$, implies that larger containers allow us to remove smaller particles, which makes intuitive sense: we get a longer residence time. So we need only pick the largest possible container, either the `HIGH CUBE 40'` or the `Standard 40'`, both have $W = 2.35$m and $L = 12.036$m; pick which ever is cheaper. Note that the other containers have similar footprint of 28.3 m^2, so really, any one would do.
- Finally, the g in the denominator makes sense: if we could increase this, we could retrieve smaller and smaller particles for a given unit. This is the principle for using centrifuges and cyclones, which allow as to capture solids at the same throughput, but in a far smaller unit.

A plot of this minimum particle size against volumetric flow is shown below, indicating we can capture particles of 50 to 200 μm or larger, depending on the required flow rate.



A final check on Reynolds number is required, which is shown below for selected flow rates. While we are just outside the Stokes' region for part of the plot, the deviation is drag coefficient is minor, especially given our over design factor of 10. In practice I would iterate until convergence to double check my final design, but only once we have selected a more certain particle size/flow rate combination; this is just a preliminary design.

Q [m^3.s^{-1}]	D_p [μm]	v_{TSV} [m.s^{-1}]	Re	$C_{\text{D,Stokes}}$	$C_{\text{D,true}}$
0.1	30.0	0.035	0.070	338	338
1	95.0	0.354	2.244	10.6	13.4
2	134	0.709	6.349	3.77	5.76
3	164	1.063	11.66	2.05	3.69
4	189	1.418	17.95	1.33	2.75

Question 5

Provide 3 examples where cyclones are used in industrial practice. Please cite your references for this question.

Would a cyclone have been an better option for the previous question? Please explain.

Solution

Used in mining, saw mills, any where with dust separation: e.g. cement industry. They are also effectively used for liquid-solid separations, where they are sometimes called hydrocyclones.

A cyclone would certainly be a better option: the higher rotational forces would be many times that of gravity, implying the same separation can be achieved in a much smaller unit, saving on real-estate costs.

END