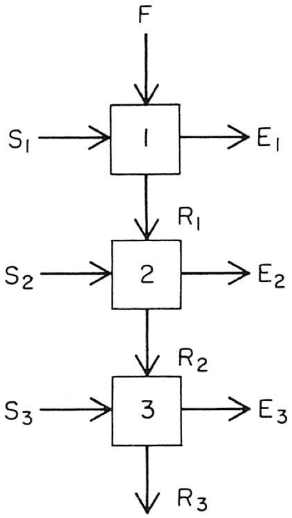
# Recap: Cross-flow arrangements

$N = 3$ in this illustration




- Recovery = fraction of solute recovered

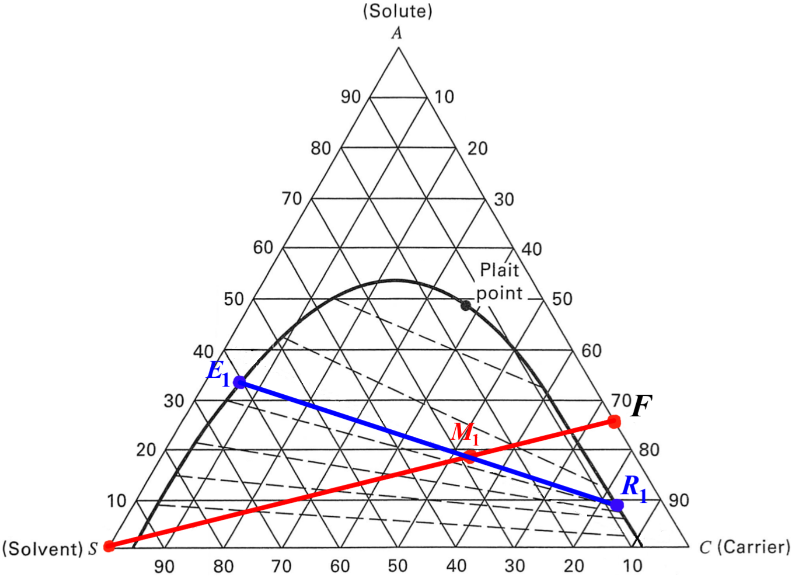
$$1 - \frac{(x_{R_N})(R_N)}{(x_F)(F)}$$

- Concentration of overall extract = solute leaving in each extract stream, divided by total extract flow rate

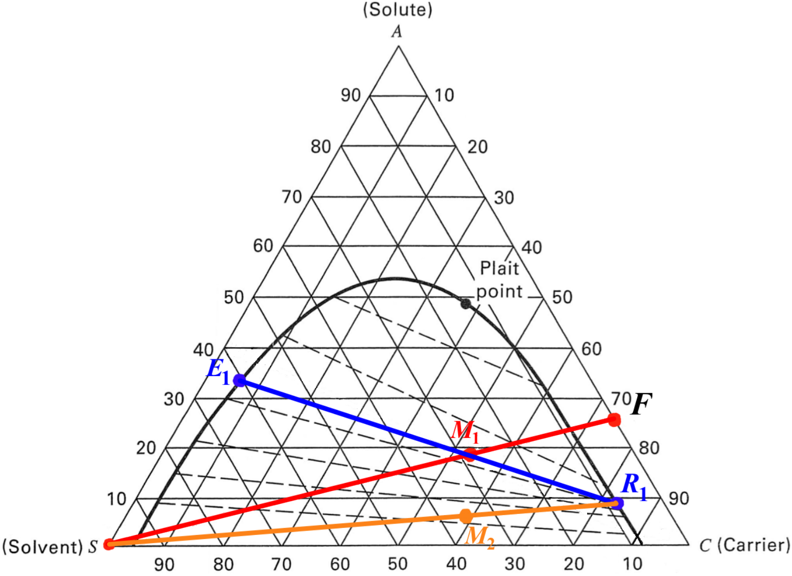
$$\frac{\sum_n^N (y_{E_n})(E_n)}{\sum_n^N E_n}$$

[Schweitzer, p 1-263]

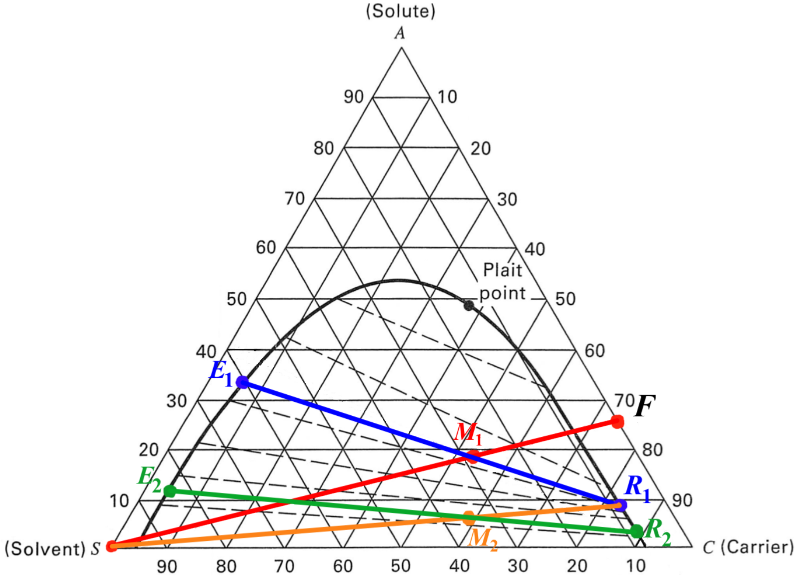
# Review from last time



# Review from last time

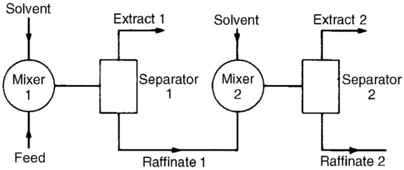


# Review from last time



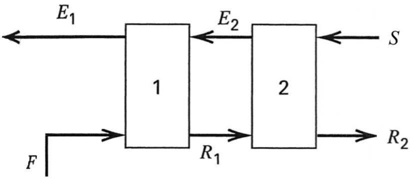
# Cross-current *vs* counter-current

## Cross-current ($N = 2$ stages)




- We combine multiple extract streams
- (Only 2 in illustration)
- In general: $y_{E_1} > y_{E_2} > \dots$
- Fresh solvent added at each stage

## Counter-current ($N = 2$ stages)




- "Re-use" the solvent, so
- Far lower solvent flows
- Recovery $= 1 - \dfrac{(x_{R_N})(R_N)}{(x_F)(F)}$
- Concentration $= y_{E_1}$
- *How many stages? What solvent flow?*

You will have an assignment question to compare and contrast these two configurations

# What we are aiming for

General approach:

1. Use ternary diagrams to determine operating lines
2. Estimate number of "theoretical plates" or "theoretical stages"
3. Convert "theoretical stages" to actual equipment size. E.g. assume we calculate that we need $N \approx 6$ theoretical stages.
   - does **not mean** we require 6 mixer-settlers (though we could do that, but costly)
   - it means we need a **column** which has equivalent operation of 6 counter-current mixer-settlers that fully reach equilibrium
   - at this point we resort to correlations and vendor assistance
   - vendors: provide HETS = height equivalent to a theoretical stage
   - use that to size the column

- unit height (or size) $= \dfrac{\text{HETS} \times \text{number of theoretical stages}}{\text{stage efficiency}}$

# For example

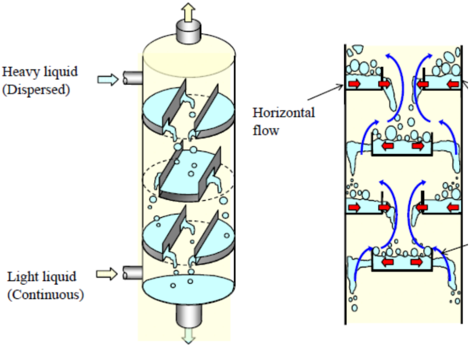
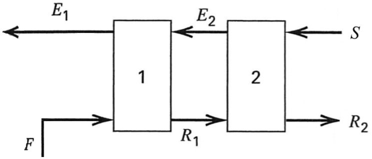




Figure 2: Concept and Flow of WINTRAY System

[WINTRAY (Japanese company; newly patented design)]

# Two *counter-current* units

*Reference for this section*: Seader textbook, 3rd ed, p 312 to 324.




**Consider $N = 2$ stages** for now. Steady state mass balance:

$$F + E_2 = R_1 + E_1 \qquad\qquad R_1 + S = R_2 + E_2$$

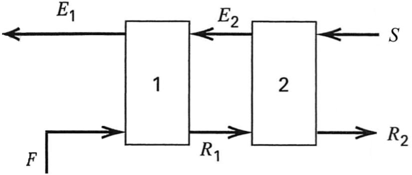
*Rearrange*:

$$F - E_1 = R_1 - E_2 \qquad\qquad R_1 - E_2 = R_2 - S$$

$$(F - E_1) = (R_1 - E_2) = (R_2 - S) = \boxed{P}$$

Note: each **difference** is equal to $P$ (look on the flow sheet above where those *differences* are).

# Counter-current graphical solution: 2 units




Rearranging again:

$$F = E_1 + P$$

$$R_1 = E_2 + P$$

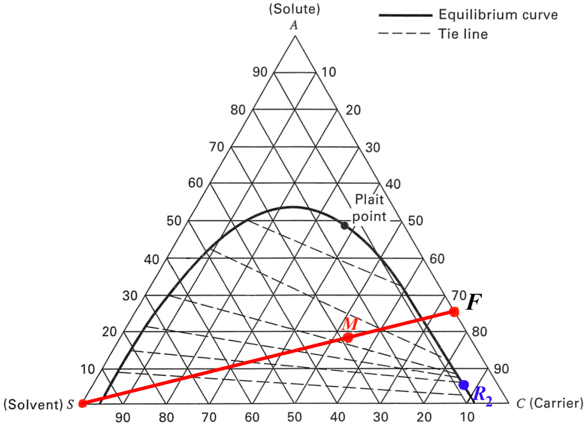
$$R_2 = S + P$$

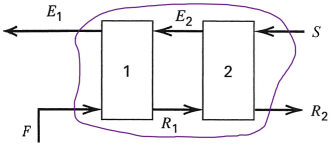
*Interpretation*: $P$ **is a fictitious operating point on the ternary diagram** (from lever rule)

- $F$ is on the line that connects $E_1$ and $P$
- $R_1$ is on the line that connects $E_2$ and $P$
- $R_2$ is on the line that connects $S$ and $P$

# Counter-current graphical solution: 2 units

**Step 1**




| Feed | Solvent |
|---|---|
| $F = 250$ kg | $S = 100$ kg |
| $x_{F,A} = 0.24$ | $x_{S,A} = 0.0$ |
| $x_{F,C} = 0.76$ | $x_{S,C} = 0.0$ |
| $x_{F,S} = 0.00$ | $x_{S,S} = 1.0$ |

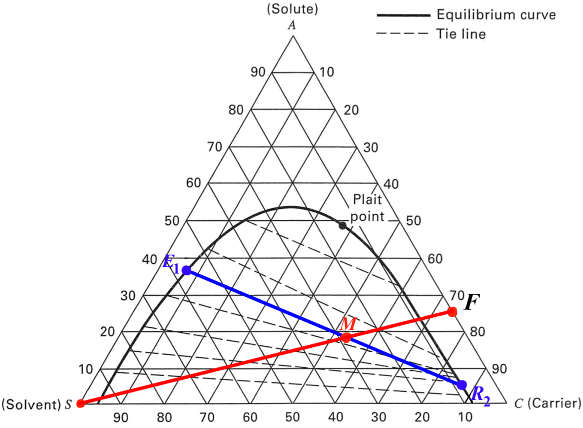
Overall balance gives:

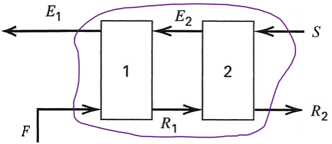
$$M = S + F = E_1 + R_2$$

For example, let's require $x_{R_2,A} = 0.05$ (solute concentration in raffinate).
Given an $S$ flow rate, what is $y_{E_1,A}$? (concentration of solute in extract)

# Counter-current graphical solution: 2 units

**Step 2**



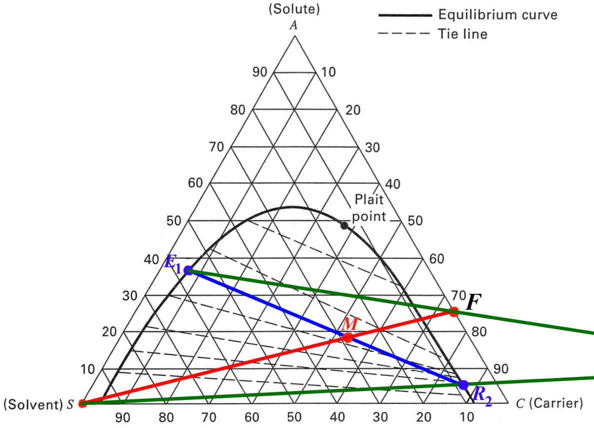



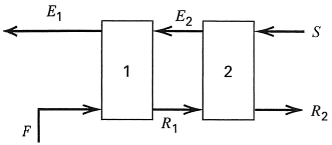

**Note**: the line connecting $E_1$ **to** $R_2$ **is not a tie line**. We use the lever rule and an overall mass balance $(F + S = E_1 + R_2)$ to solve for all flows and compositions of $F, S, E_1$, and $R_2$.

$y_{E_1,A} \approx 0.38$ is found from an overall mass balance, through $M$. Simply connect $R_2$ and $M$ and project out to $E_1$.

# Counter-current graphical solution: 2 units

**Step 3**




**Recall**:

$$F = E_1 + P$$

$F$ is on the line that connects $E_1$ and $P$

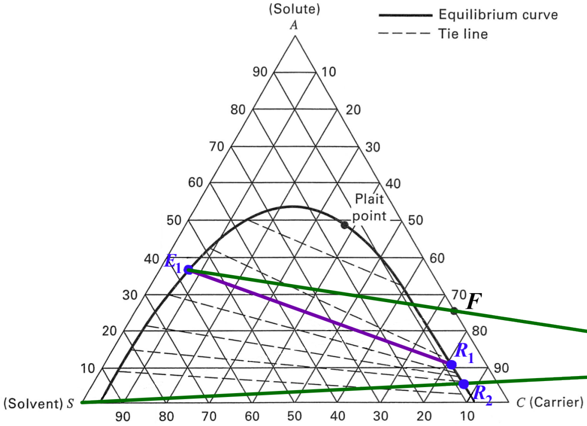
$$R_2 = S + P$$

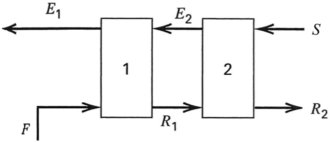
$R_2$ is on the line that connects $S$ and $P$

Extrapolate through these lines until intersection at point $P$.

# Counter-current graphical solution: 2 units

**Step 4**

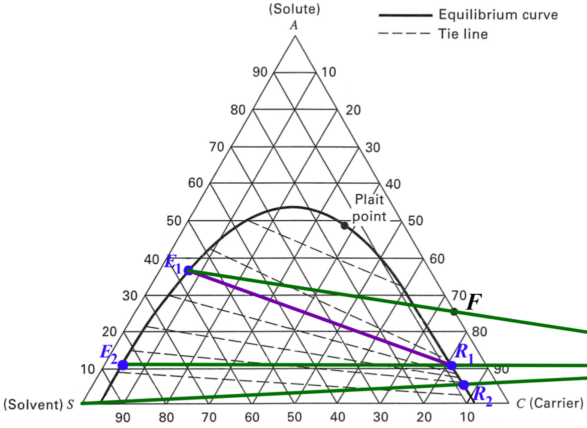


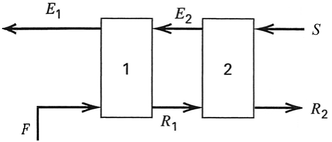




Once we have $E_1$, we can start: note that in stage 1 the $R_1$ and $E_1$ streams leave in equilibrium and can be connected with a tie line.

# Counter-current graphical solution: 2 units

**Step 5**




**Again recall**:

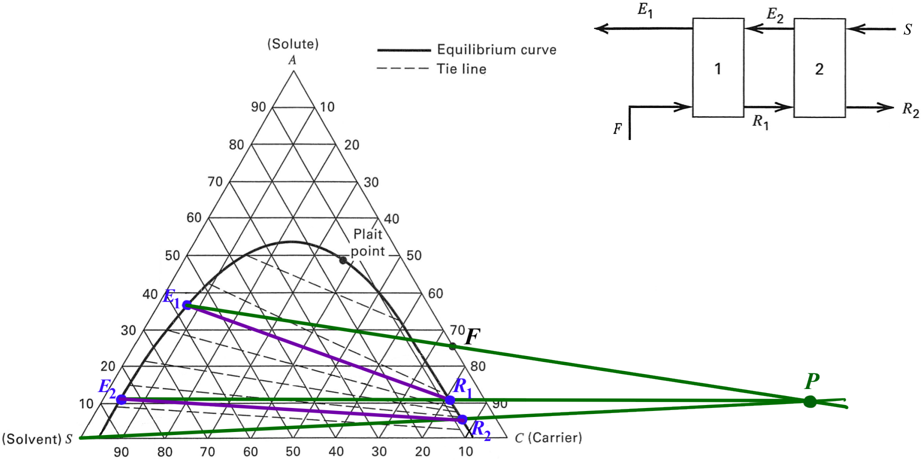
$$R_1 = E_2 + P$$

$R_1$ is on the line that connects $E_2$ and $P$

Since we have point $P$ and $R_1$ we can bring the operating line back and locate point $E_2$

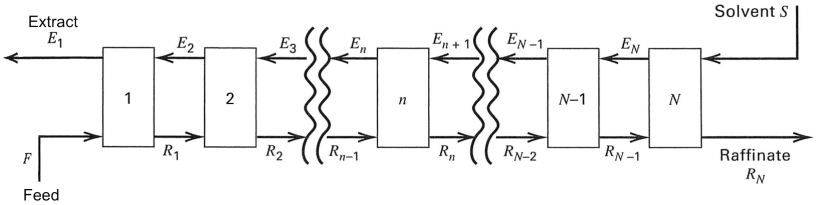
# Counter-current graphical solution: 2 units

**Step 6**




The last unit in a cascade is a special case: we already know $R_{N=2}$, but we could have also calculated it from the tie line with $E_2$. We aim for some overshoot of $R_N$. (Good agreement in this example.)

# In general: *Counter-current* units




$$F + E_2 = E_1 + R_1 \qquad E_2 + R_2 = E_3 + R_1 \qquad E_n + R_n = E_{n+1} + R_{n-1}$$

*Rearrange*:

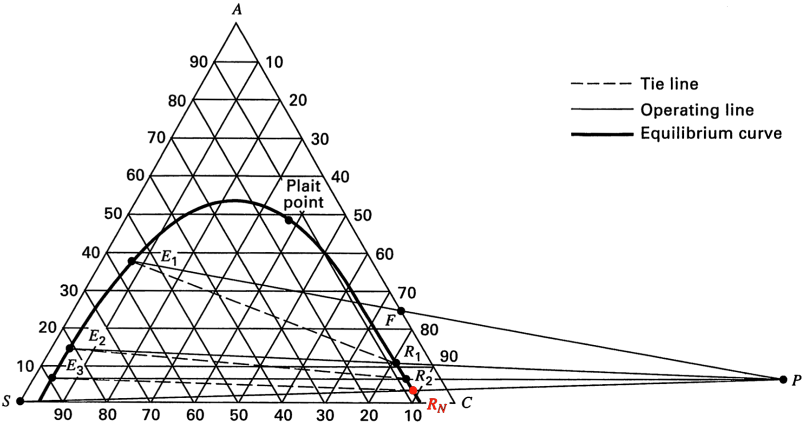
$$F - E_1 = R_1 - E_2 \qquad R_1 - E_2 = R_2 - E_3 \qquad R_{n-1} - E_n = R_n - E_{n+1}$$

$$(F - E_1) = (R_1 - E_2) = \ldots = (R_{n-1} - E_n) = (R_n - E_{n+1}) = \ldots = (R_N - S) = \mathbf{P}$$

Notes:

1. each difference is equal to $P$ (the difference between flows)
2. $E_n$ and $R_n$ are in equilibrium, leaving each stage [via tie line]

# Counter-current graphical solution



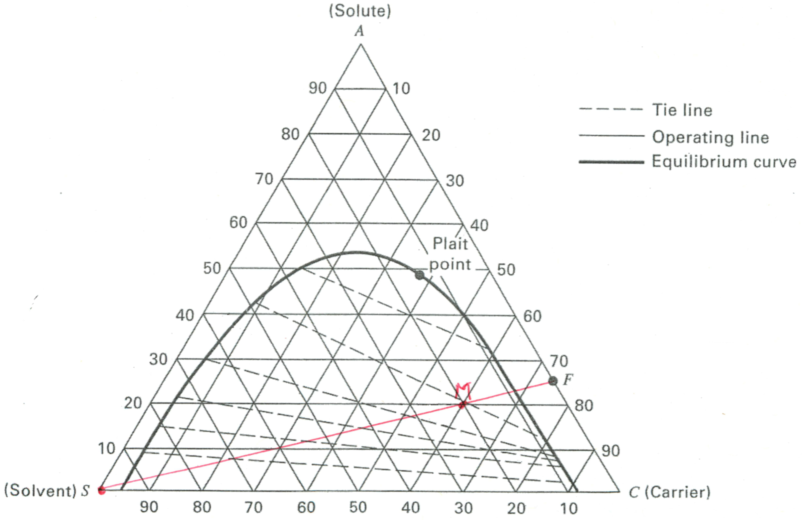

1. We know $F$ and $S$; connect with a line and locate "mixture" $M$
2. Either specify $E_1$ or $R_N$ (we will always know one of them)
3. Connect a straight line through $M$ passing through the one specified
4. Solve for unspecified one [via tie line]
5. Connect $S$ through $R_N$ and extrapolate
6. Connect $E_1$ through $F$ and extrapolate; cross lines at $P$
7. Locate $P$ by intersection of 2 lines
8. In general: connect $E_n$ and $R_n$ via equilibrium tie lines

# Tutorial-style question

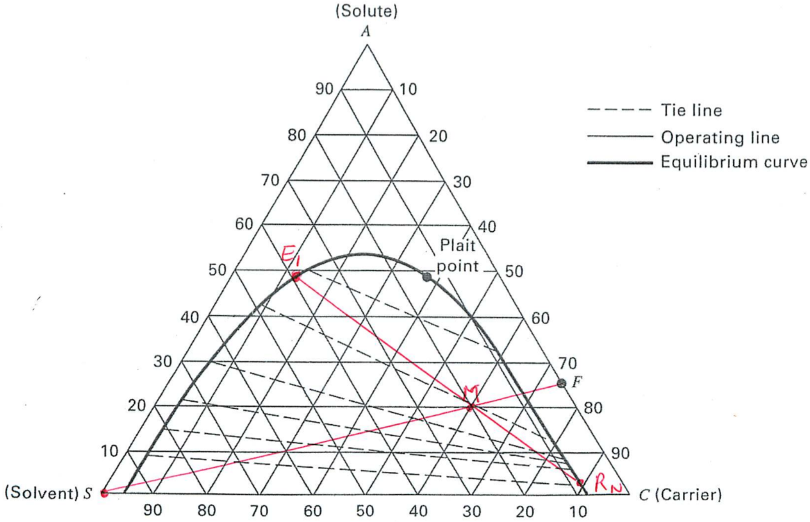
Consider a system for which you have been given the ternary diagram (see next slides). $A$ = solute, $S$ = solvent (100% pure), $C$ = carrier. The feed, $F$ enters at 112 kg/hr with composition of 25 wt% solute and 75 wt% carrier.

1. Calculate the flow and composition of the extract and raffinate from:
   - 1st cross-current stage, using a pure solvent flow of 50 kg/hr.
   - 2nd cross-current stage, with an additional solvent flow of 50 kg/hr.
2. For the overall 2-stage cross-current system, find the:
   - overall recovery [*answer:* ∼93%]
   - overall concentration of combined extract streams [*answer:* ∼21%]
3. The objective now is to have a counter-current system so the raffinate leaving in the $N^{\text{th}}$ stage, $R_N$ has $y_{R_N} = 0.025$
   - Show the construction on the ternary diagram for the number of equilibrium stages to achieve $x_{R_N} = 0.025$, given a solvent flow of 28 kg/hr.
   - Calculate the overall recovery and concentration of the extract stream.
   - Plot on the same axes the concentrations in the extract and raffinate streams.

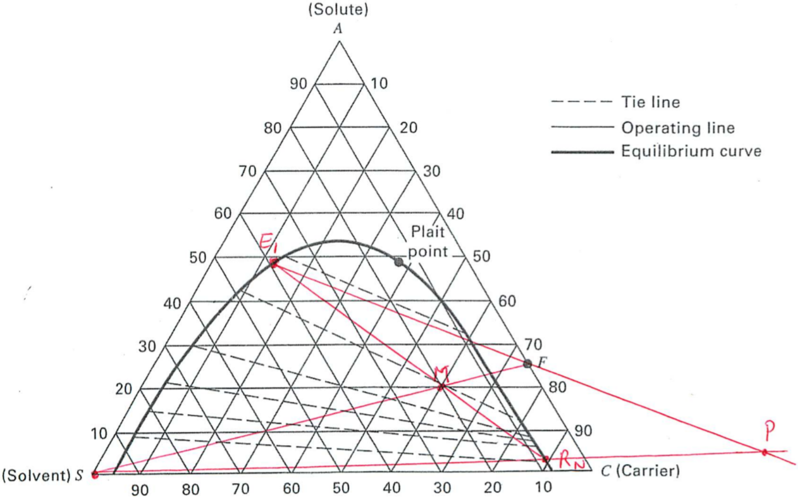
# Tutorial solution: step 1



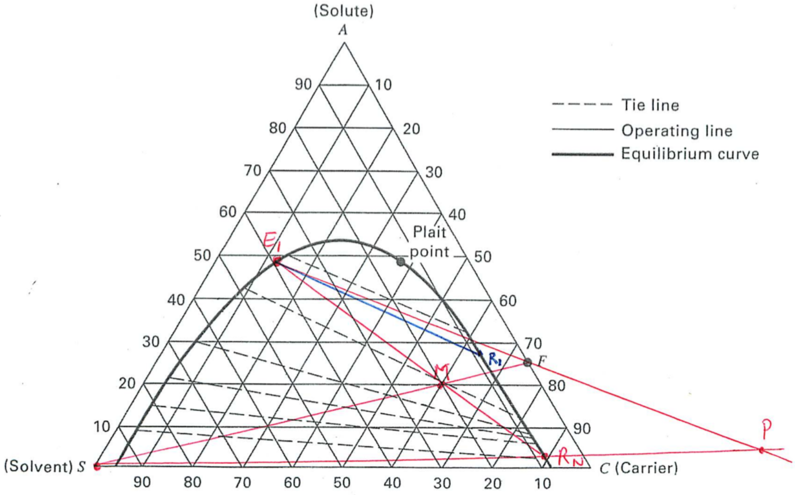
# Tutorial solution: step 2



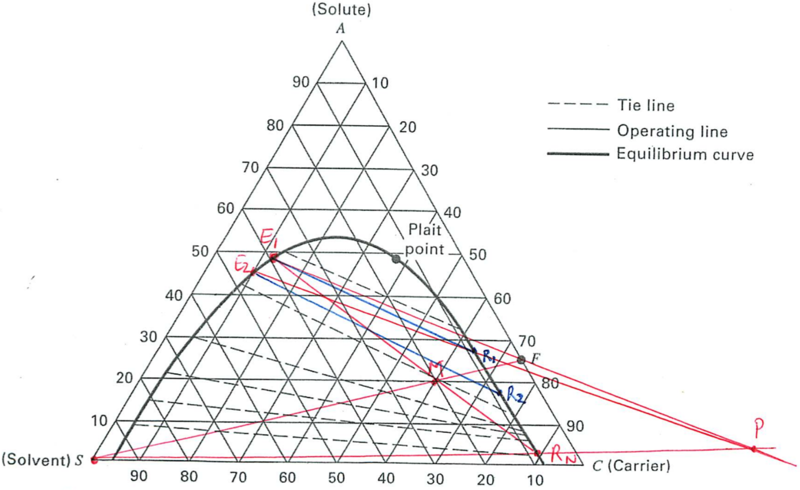
# Tutorial solution: step 3



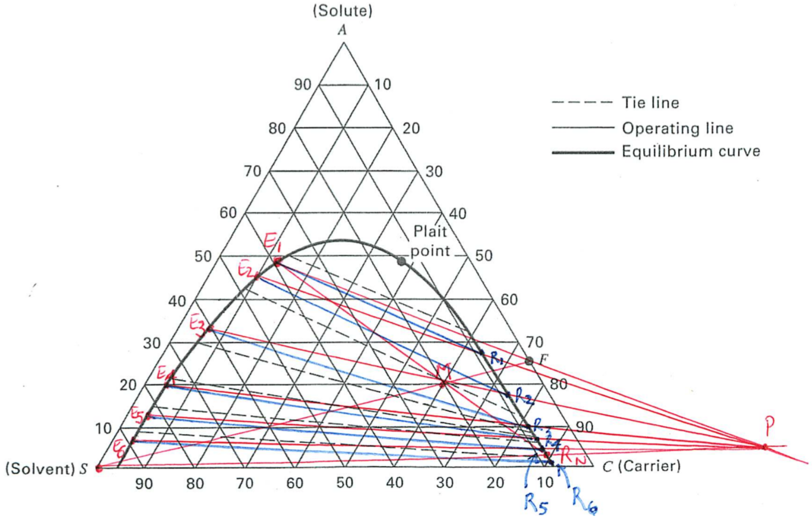
# Tutorial solution: step 4



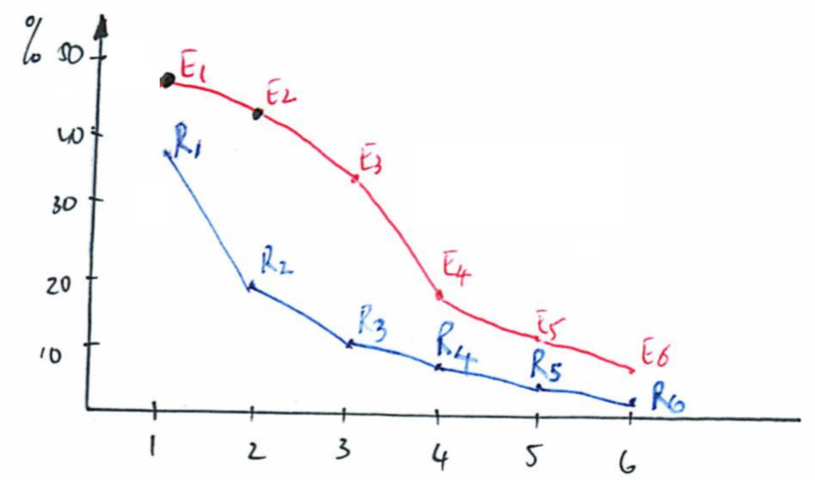
# Tutorial solution: step 5



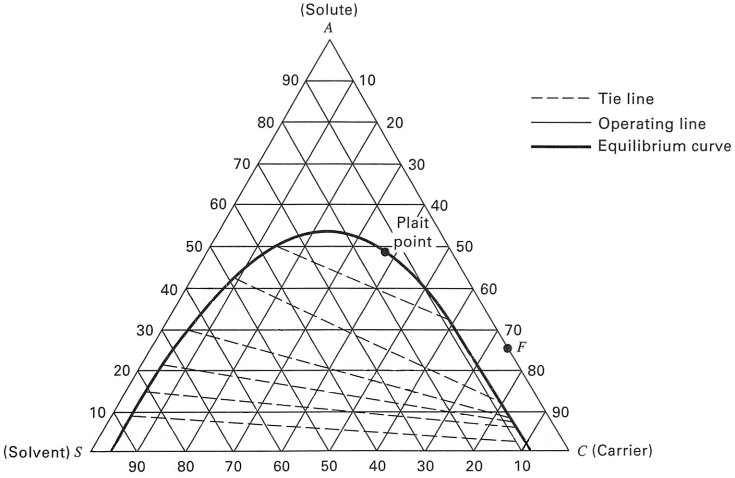
# Tutorial solution: step 6



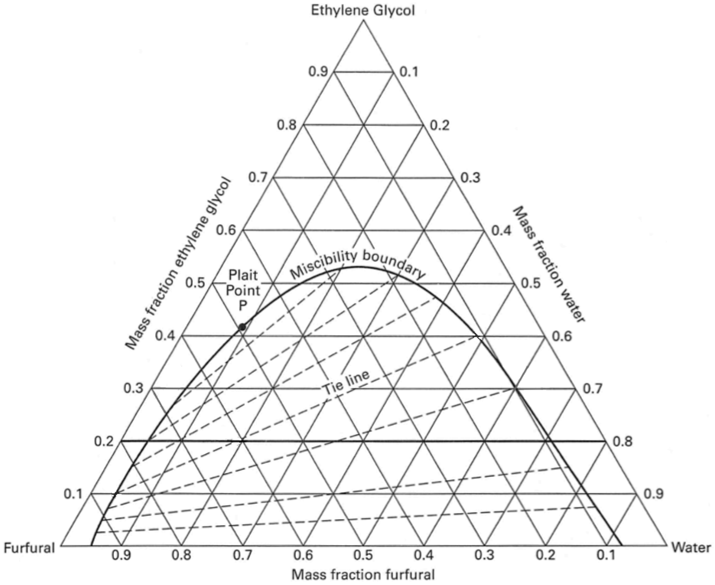
# Tutorial solution: concentration profile



# For practice (A)

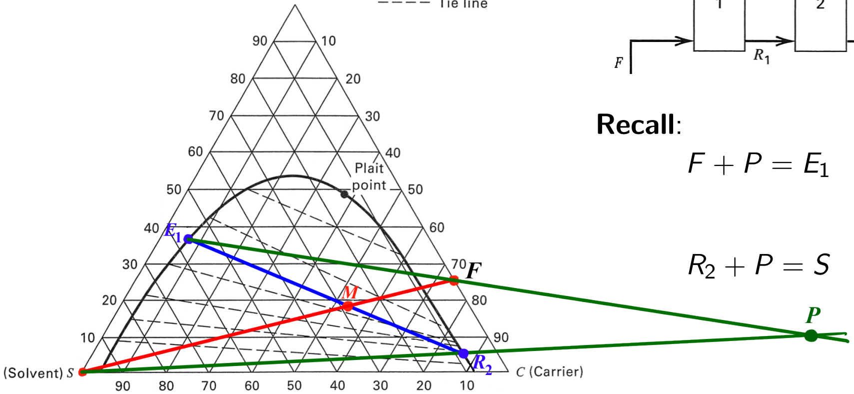


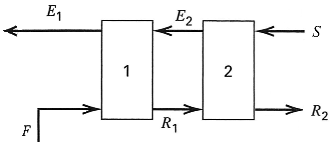
# For practice (B)



# Counter-current graphical solution: 2 units

**Step 3(b)**




**Recall**:

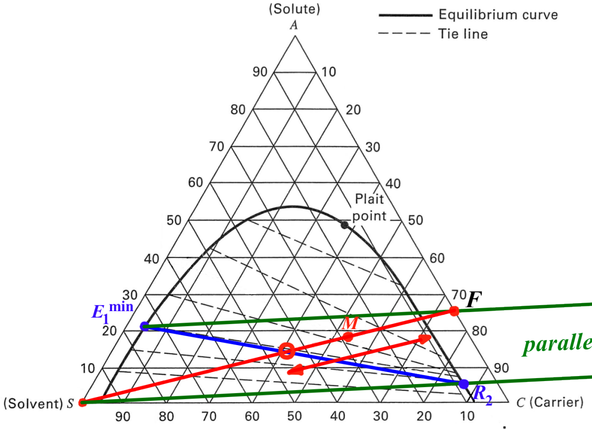
$$F + P = E_1$$

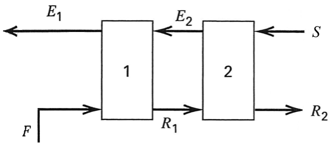
$$R_2 + P = S$$

**Thought experiment**: What is the minimal achievable $E_1$ concentration? *mentally move point* *M* *towards* $S$. What happens to $P$ as solvent flow $S$ is increased? Alternative explanation next.

# Counter-current graphical solution: maximum solvent flow

**Step 3(b)**




**Recall**:

$$F + P = E_1$$

$$R_2 + P = S$$

Subtle point: minimal achievable $E_1^{min}$ concentration:

- occurs at a certain *maximum* solvent flow rate indicated by ○
- note that $R_2$ is fixed (specified) in this example