

# Separation Processes, ChE 4M3, 2013

## Assignment 4

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**Objectives:** Understanding membrane separation processes and some practice with liquid-liquid extraction systems.

### Question 1 [5]

An microfiltration membrane operating with pure feed of water produces a flux of  $0.05 \text{ kg}\cdot\text{s}^{-1}\cdot\text{m}^{-2}$  when operated with an applied pressure different of 25 kPa.

1. What is the resistance component due to the membrane? Specify the units.
2. If operated with a protein-water mixture at a 22 kPa pressure difference, a flux of  $242 \times 10^{-6} \text{ kg}\cdot\text{s}^{-1}\cdot\text{m}^{-2}$  is measured at steady state. What is the resistance due to cake build-up? Specify the units.

*Solution*

The general equation is:

$$J = \frac{\Delta P}{\mu (R'_c + R'_m)}$$

1. When pure water is being filtered, the resistance due to cake build up is zero, so solve the equation for  $R'_m$ . Using/assuming water viscosity to be  $\mu = 1 \times 10^{-3} \text{ Pa}\cdot\text{s}$ , then  $R'_m = 5 \times 10^8 \text{ m}^2\cdot\text{kg}^{-1}$ .
2. This membrane resistance remains the same when filtering the protein, so solving the general equation gives  $R'_c = 9.2 \times 10^{10} \text{ m}^2\cdot\text{kg}^{-1}$ .

### Question 2 [25]

An ultrafiltration membrane is to be used to separate paint from a liquid stream, and to achieve a more concentrated paint-water mixture. You plan to use several [Zeeweed 1500 membrane modules](#) (download the fact sheet and watch the video). The supplier has given you the performance equation, based on some joint test work on their experimental system:

$$J_v = 0.042 \ln \left( \frac{27}{C} \right)$$

where the bulk concentration  $C$  has units of  $\text{kg}\cdot\text{m}^{-3}$  and flux is measured in  $\text{m}^3\cdot\text{hour}^{-1}\cdot\text{m}^{-2}$ .

The feed waste stream arrives at a flow rate of  $5.1 \text{ m}^3\cdot\text{hour}^{-1}$  with concentration of  $0.6 \text{ kg}\cdot\text{m}^{-3}$ . The aim is to achieve a paint-water concentration leaving the membrane of  $20 \text{ kg}\cdot\text{m}^{-3}$ , so that it can be reused.

1. What is the flow rate of the cleaned water from the system? (We might be able to use it elsewhere in our process)
2. What number of modules would we have to purchase if operated in a single-stage feed-and-bleed configuration?

3. We could consider two membrane systems in series, where each system can have a different area. How many modules should you use in system 1 and how many modules in system 2, so that the total number of modules you buy are at a minimum?
4. For this 2 systems in series arrangement, what will be the total flow of cleaned water (*hint*: think carefully!)
5. What are your thoughts on putting four stages in series?

*Please note*: it is in your interest to solve this problem as if it were in a test or exam; i.e. don't use computer software. Solve part 3 on a computer after you've done some guess-and-check by hand.

### Solution

1. A single membrane unit is being used: the aim is calculate the total area provided by the parallel modules. As shown in the solutions developed in class:
  - From a mass balance around the unit, assuming no paint leaves in the permeate (a reasonable assumption for ultrafiltration):  $Q_0 C_0 = Q_{R1} C_{R1}$
  - From a volume balance we have  $Q_0 = Q_{P1} + C_{R1}$
  - The given flux equation  $J_v = \frac{Q_{P1}}{A_1} = 0.042 \ln \left( \frac{27}{C} \right)$
  - When solving the above equations we find that:  $Q_{R1} = \frac{Q_0 C_0}{C_{R1}} = \frac{(5.1)(0.6)}{20} = 0.153 \text{ m}^3 \cdot \text{hr}^{-1}$

So the flow of clean water from the single unit is  $Q_{P1} = 4.947 \text{ m}^3 \cdot \text{hr}^{-1}$ .

2. Use the flux equation and solve for  $A_1 = 392.5 \text{ m}^2$ , provided by  $\frac{392.5}{55.7} = 7.05 \approx 8$  modules.
3. We require to find the areas of the two units to determine this. As with the single module system, set up the mass and volume balances for both units. Substitute these into the flux equation for each unit:

$$\begin{aligned}
 Q_0 &= Q_{P1} + Q_{R1} \\
 3.0 &= 0.042 A_1 \ln \left( \frac{27}{C_{R1}} \right) + \frac{3.06}{C_{R1}} \\
 Q_{R1} &= Q_{P2} + Q_{R2} \\
 \frac{3.06}{C_{R1}} &= 0.42 A_2 \ln \left( \frac{27}{C_{R2}} \right) + \frac{3.06}{20}
 \end{aligned}$$

These 2 equations involve 3 unknowns:  $A_1$ ,  $A_2$  and  $C_{R1}$ . As is the case with optimization, there are always spare degree(s) of freedom to adjust. In this case we adjust  $C_{R1}$  to different values and find the result of  $A_1$  and  $A_2$  so their total sum is at a minimum.

Some values that were tried:

| $C_{R1}$ [kg.m <sup>-3</sup> ] | $A_1$ in m <sup>2</sup> | $A_2$ in m <sup>2</sup> | $A_1 + A_2$ in m <sup>2</sup> |
|--------------------------------|-------------------------|-------------------------|-------------------------------|
| 2                              | 32.7 [1 modules]        | 109 [2 modules]         | 3 modules                     |
| 3                              | 44.2 [1 modules]        | 68.8 [2 modules]        | 3 modules                     |
| 4                              | 54.1 [1 modules]        | 48.6 [1 module]         | 2 modules (minimum)           |
| 5                              | 63.4 [2 modules]        | 36.4 [1 module]         | 3 modules                     |

This tables makes sense: as we increase the intermediate retentate concentration,  $C_{R1}$ , the area required in the first unit to achieve that concentration increases, meaning that less area is required in the second unit.

There will be a broad range of valid solutions around  $C_{R1} = 4 \text{ kg.m}^{-3}$ , corresponding to 1 module in the first unit and 1 module in the second unit.

- Let the optimal arrangement be the third row in the table (note there are many possible, similar solutions), then the total flow of cleaned water is found by calculating the permeate flows from both units.

Note however, that the area of the first unit,  $54.1 \text{ m}^2$ , is not actually achieved, it is  $A_1 = 55.7 \text{ m}^2$ . So solve the membrane mass and volume balance equations for this new area to calculate  $C_{R1}$  (requires an iterative solution). Then calculate the permeate flow rate  $Q_{P1} = 0.042A_1 \ln\left(\frac{27}{C_{R1}}\right)$  for  $A_1 = 55.7 \text{ m}^2$  and  $Q_{P1} = 4.367 \text{ m}^3.\text{hr}^{-1}$ .

Repeat this for the second unit using  $A_2 = 55.7 \text{ m}^2$  to calculate  $Q_{P2} = 0.587 \text{ m}^3.\text{hr}^{-1}$ . Sum the two for the final solution of clean water flow,  $Q_{P \text{ total}} = 4.954 \text{ m}^3.\text{hr}^{-1}$ .

Note that  $C_{R2}$  will exceed the specifications of  $20 \text{ kg.m}^{-3}$  from the overall system, it will be  $21.01 \text{ kg.m}^{-3}$ .

- Using more than 2 feed-plus-bleed units will be counterproductive for more stages. We are already using one module per stage, as such, further treatment is not necessary.

### Question 3 [15]

We are testing a RO membrane at  $25^\circ\text{C}$ . The feed contains  $0.5 \text{ wt}\%$  NaCl at a flow rate of  $1250 \text{ L/hour}$ . The retentate is at  $1.4 \text{ wt}\%$  NaCl and the permeate at  $0.012 \text{ wt}\%$ . The pressure drop applied is  $15 \text{ atmospheres}$ . When using pure water, the permeate flow is  $1000 \text{ L/hour}$  at the same pressure drop.

*Note:* the osmotic pressure difference can be approximated from the difference between the retentate and permeate concentrations (initially we used the feed minus permeate concentrations when we didn't have the retentate concentration). The reality is the osmotic pressure difference should be calculated from values between the feed and retentate concentrations, integrated as it changes along the the membrane area.

- Find the outlet flow rates of the permeate and the retentate.
- What is the numerical value of the "cut"?
- What is the separation factor's value for this system?

#### Solution

*After using this question from a textbook, I discovered it was over-specified. The right information wasn't present to solve the problem in the intended manner. The grading for this question will be pass/fail, if you've made a reasonable attempt at answering. Please check the solution below though. Ideally the osmosis equations should have been used. See the next question for an example of that.*

- The concentrations can be expressed in  $\text{g/L}$ ; for example, for the retentate,  $1.4 \text{ wt}\% = \frac{1.4}{100 - 1.4} \times \frac{1000\text{g}}{1\text{L}} = C_R = 14.2 \text{ g/L}$ . Similarly,  $C_P = 0.12 \text{ g/L}$ .

From a volume balance,  $Q_P = Q_0 - Q_R = 1250 - Q_R \text{ L/hr}$ ; then using the mass balance on the solute, we can solve for  $Q_R = 435 \text{ L/hour}$  and  $Q_P = 1250 - 435 = 815 \text{ L/hour}$ .

- The cut  $= \frac{Q_P}{Q_0} = \frac{815}{1250} = 0.65$

3. The separation factor =  $S_{ij} = \frac{x_{i,1}/x_{j,1}}{x_{i,2}/x_{j,2}} = \frac{1.4/(100 - 1.4)}{0.012/(100 - 0.0012)} = \frac{1.4/(100 - 1.4)}{0.00012} = \frac{14.2}{0.12} = 118$ .

**Question 4 [15 = 2 + 2 + 2 + 4 + 2 + 1 + 2]**

*Final exam, 2012*

Reverse osmosis of a 50 g.L<sup>-1</sup> sucrose-water solution is being experimentally investigated. The osmotic pressure in a sucrose-water system is approximated by  $\pi = 0.054m$ , where  $m$  is the sucrose concentration in g.L<sup>-1</sup>, and  $\pi$  has units of atmospheres. The membrane is operated so the cut is 40%, with a feed rate of 0.25 m<sup>3</sup>.hr<sup>-1</sup>. Using a pressure difference of 30 atm, the permeance of the membrane with respect to the solvent is  $5.4 \times 10^{-4}$  kg.s<sup>-1</sup>.m<sup>-2</sup>.atm<sup>-1</sup>. For reference, the molar mass of water is 18 g.mol<sup>-1</sup>, and for sucrose it is 342 g.mol<sup>-1</sup>.

If the retentate concentration leaving the system is measured as 81.8 g.L<sup>-1</sup> of sucrose, calculate the following other quantities:

1. The retentate flow rate leaving the membrane. [2]
2. The permeate concentration in g.L<sup>-1</sup>. [2]
3. The osmotic pressure difference across the membrane. [2]
4. The solvent (water) flux expressed in LMH. [4]
5. The solute (sucrose) **mass** flux expressed on an hourly basis. [2]
6. The rejection coefficient. [1]
7. The separation factor. [2]

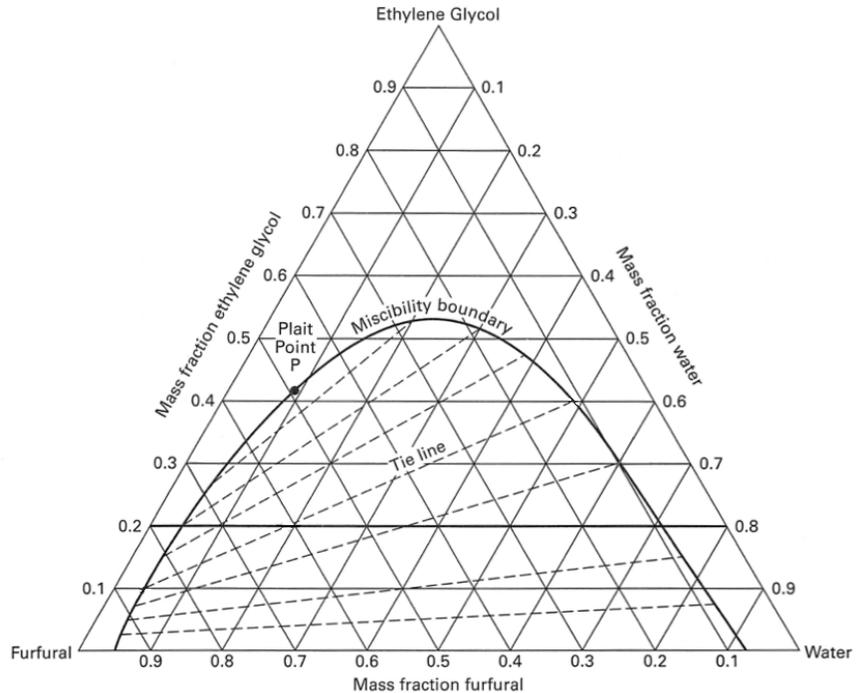
*Solution*

Show a diagram with the notation. Rough calculations and final answers are given. Unit conversions are required along the way.

1.  $Q_P = \theta Q_F = 0.4 \times 0.25 = 0.1 \text{ m}^3.\text{hr}^{-1}$ , so  $Q_R = Q_F - Q_P = 0.25 - 0.1 = 0.15 \text{ m}^3.\text{hr}^{-1}$ .
2. From an overall mass balance  $C_P = \frac{Q_F C_F - Q_R C_R}{Q_P} = 2.3 \text{ g.L}^{-1}$
3.  $\Delta p_i = \pi_R - \pi_P = 0.054(81.8 - 2.3) = 4.29 \text{ atm}$ , this is an overestimate, since  $\pi_R$  is at the retentate side, where the osmotic pressure is the highest. The feed point could also be used, or more accurately, it should be integrated over the membrane.
4. Solvent flux =  $J_{v,\text{solv}} = A_{\text{solv}}(\Delta P - \Delta\pi) = 1.39 \times 10^{-5} \frac{\text{m}^3 \text{ solvent}}{\text{m}^2.\text{s}}$   
 Converted to LMH: it is 49.9 LMH.
5.  $J_{\text{solute}} = J_{\text{solv}} C_P = 0.115 \frac{\text{kg}}{\text{m}^2.\text{hr}}$
6.  $R = 1 - \frac{C_P}{C_F} = 1 - \frac{2.3}{50} = 0.95$ .
7.  $S_{ij} = \frac{x_{i,1}/x_{j,1}}{x_{i,2}/x_{j,2}} = \frac{81.8}{2.3} = 35.6$ .

### Question 5 [20]

The aim is to recover ethylene glycol from water using furfural as solvent (the ternary diagram for furfural, ethylene glycol and water at the unit's operating temperature is provided).



- Calculate the equilibrium compositions leaving a single mixer-settler unit operated at:
  - the recycled solvent stream:  $120 \text{ kg}\cdot\text{hr}^{-1}$  with a composition of 80% furfural, and unrecovered ethylene glycol at 5%, with the rest being water.
  - the feed stream contains 40% ethylene glycol and 60% water at a rate of  $200 \text{ kg}\cdot\text{hr}^{-1}$ .
- Calculate the flow rates of the extract and raffinate streams.
- What is the distribution (partition) coefficient,  $D_A$ , for A = ethylene glycol (see course notes for definition)?
- What is the distribution (partition) coefficient,  $D_C$ , for C = water, using this same definition?
- What is the separation factor for  $i$  = ethylene glycol from  $j$  = water in the 1=extract and 2=raffinate streams? Also show the relationship between the separation factor and the previous two distribution coefficients.

*Solution*

- From the ternary diagram constructions we obtain (approximately):

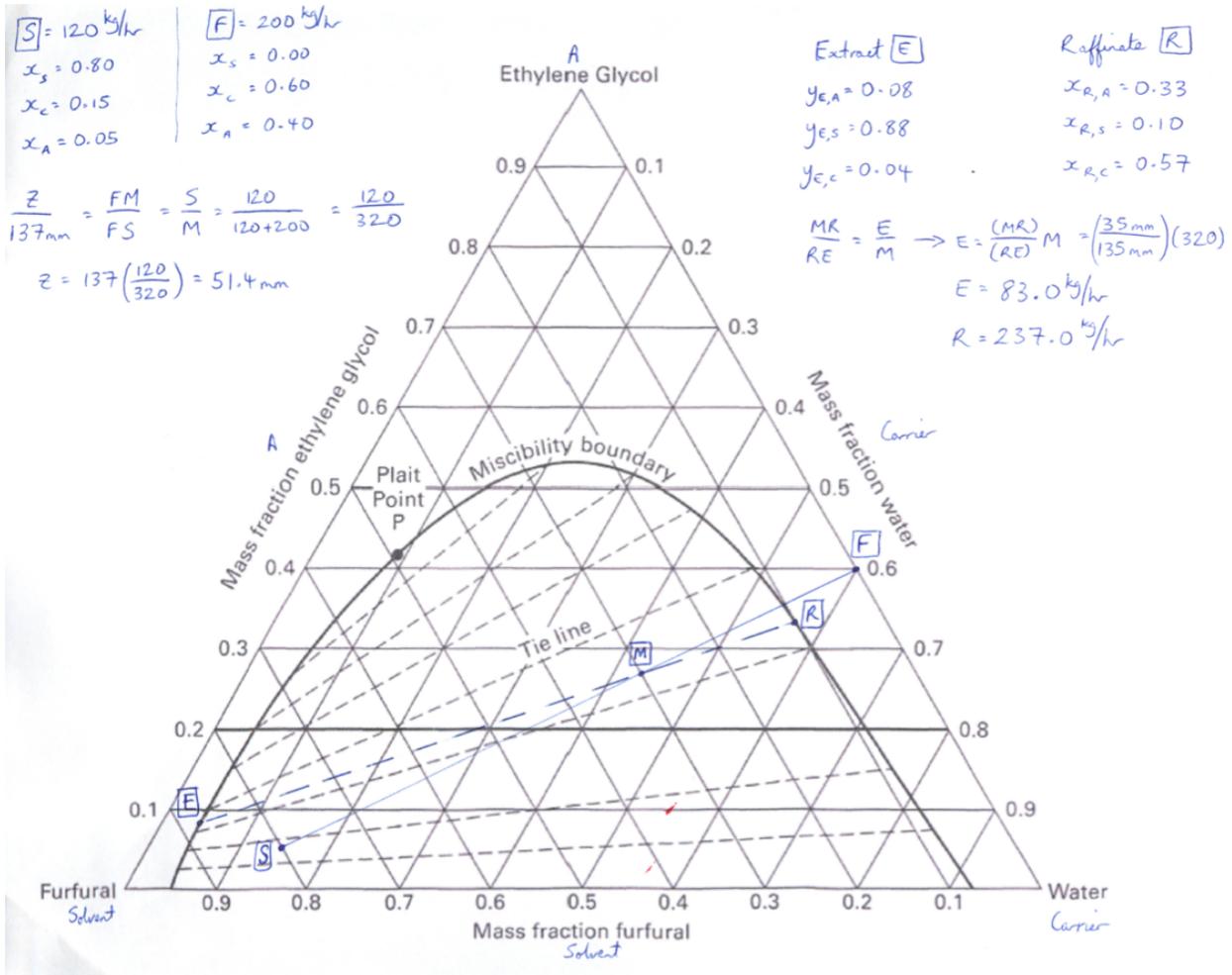
| <i>E</i>                               | <i>R</i>                                |
|----------------------------------------|-----------------------------------------|
| $y_{E,A} = 0.08$                       | $x_{R,A} = 0.33$                        |
| $y_{E,S} = 0.88$                       | $x_{R,S} = 0.10$                        |
| $y_{E,C} = 0.04$                       | $x_{R,C} = 0.57$                        |
| $E = 83 \text{ kg}\cdot\text{hr}^{-1}$ | $R = 237 \text{ kg}\cdot\text{hr}^{-1}$ |

2. See above, also calculated from the lever rule on the diagram.

3. Distribution (partition) coefficient for A = ethylene glycol =  $D_A = \frac{y_{E,A}}{x_{R,A}} = \frac{0.08}{0.33} = 0.242$

4. Distribution (partition) coefficient for C = water =  $D_C = \frac{y_{E,C}}{x_{R,C}} = \frac{0.04}{0.57} = 0.07$

5. The separation factor is  $S_{i,j} = \frac{x_{i,1}/x_{j,1}}{x_{i,2}/x_{j,2}} = S_{A,C} = \frac{x_{E,A}/x_{E,C}}{x_{R,A}/x_{R,C}} = \frac{x_{E,A}}{x_{R,A}} \cdot \frac{x_{R,C}}{x_{E,C}} = \frac{D_A}{D_C} = \frac{0.242}{0.07} = 3.45$ , which is very low, as expected from the construction on the ternary diagram.



END