Separation Processes, ChE 4M3, 2014

Assignment 3

Kevin Dunn, kevin.dunn@mcmaster.ca **Due date: 14 October 2014**

Objectives: Consideration of centrifuges, filtration, and cyclones: not only the design of these units, but also operating them under different conditions.

Question 1 [6]

1. Find the specific cake resistance for pyrite pellets that are 100 μm.
2. Is the cake resistance higher or lower for particles that are 50 μm? (interpret this, and the prior answer)
3. If your particles are not spherical, which equivalent particle size would be suitable to calculate for the purpose of filtration calculations?

Solution

1. Pyrite has density of approximately 5000 kg.m^{-3} (check various resources and use an average), and at a particle size of $d = 100 \times 10^{-6}$ m, the specific cake resistance is $\alpha = \dfrac{(k_1)(1-\epsilon)(S_0^2)}{(\epsilon)^3(\rho_p)} = \dfrac{(4.17)(1-0.4)(\frac{6}{d})^2}{(0.4)^3(5000)} = \dfrac{0.2815}{d^2}$

   assuming $\epsilon \approx 0.4$, which is about right for spherical particles, depending on the assumption of how the particles are laid down (loose random packing is close enough to what is happening during filtration).

   The specific area per unit volume $= \dfrac{6}{d}$, which you can easily prove to yourself for spheres is $S = \dfrac{\text{area of sphere}}{\text{volume of sphere}} = \dfrac{3}{r} = \dfrac{6}{d}$.

   So the specific cake resistance is $\alpha = \dfrac{0.2815}{d^2} = \dfrac{0.2815}{(100 \times 10^{-6})^2} = 2.8 \times 10^7$ m.kg^{-1}, which is a value that is in the typical range of 10^7 to 10^9 m.kg^{-1}.

2. The cake resistance is expected to be higher for smaller particles, and is seen in the prior answer. It is $\alpha = \dfrac{0.2815}{d^2} = \dfrac{0.2815}{(50 \times 10^{-6})^2} = 1.13 \times 10^8$ m.kg^{-1}.

3. The equivalent size that's most suitable: is a sphere's diameter that has the same surface area per unit volume as the non-spherical particles, since the resistance calculations are mostly related to the channel's area through which the fluid travels (look at the Carmen-Kozeny equation from which we derive the filtration equation).

Question 2 [3]

Why might perlite (a type of volcanic glass) sometimes be added to filtration slurries? Describe the mechanism that makes this work to your advantage.

Solution

Perlite is added to filtration slurries as a filter aid. When added to the slurry, the main purpose of perlite is to decrease the cake's compressibility, because it keeps voidage higher with the rigid structure of the perlite crystal. Perlite, and other filter aids, can be used a precoat on the existing filter medium to form a protective layer that protects the medium from damage, without significantly adding to the resistance (look up the term *diatomaceous earth*, for example).

The precoats also prevent very small particles from entering and permanently blocking the pores of reusable filter medium cloths. They also help the cake to quickly fall off the medium during the cleaning cycle. As these aids are inert and usually present no problem downstream once they have been added.

Question 3 [15]

Under your supervision is a continually operating process that has a disk-bowl centrifuge. You are producing a bacterial-product that has minimum monthly targets, and you are only just meeting those targets. The centrifuge is rated with a $\Sigma = 5256$ m^2 value when operated at $\omega = 5000$ rpm for separating bacteria from a bioreactor broth. The bacteria's density is 1050 kg.m^{-3} with average particle size of 30 μm; it is fed at a concentration of 20 kg.m^{-3}.

1. What is the interpretation of this Σ value?
2. Why is a disk-bowl centrifuge applicable in this company?
3. Your company wants to get better performance from the centrifuge. A colleague has suggested that if you dilute the feed you might get a higher recovery (separation factor) for the bacteria. So the plan is to install a holding tank before the centrifuge and dilute the feed 5-fold, down to a concentration of 4 kg.m^{-3}. What will be the *quantitative result* (i.e. numeric result) on protein recovery of doing this?
4. The operator wants to re-use this same centrifuge, but operated at 6200 rpm, for separating particles of density $\rho = 1250$ kg.m^{-3} and average particle diameter of 2μm. These particles were mistakenly added to a large quantity of liquid that has $\rho = 800$ kg.m^{-3} and $\mu = 0.3$ N.s.m^{-2}. Provide some guidance to the operator as to how long it will take to process this batch of 10 m^3 of liquid.

Solution

1. Σ is the area of a sedimentation vessel that would be required to achieve the same separation.
2. A disk-bowl centrifuge is applicable in this case as we are continually producing a bacteria-based product, and these types of centrifuges provide continuous, aseptic operation. The angled discs provide a very high surface area in a compact space.
3. The person might have been confused with membrane separations where a dilute feed reduces the cake resistance; or cyclones where a dilute feed stream does affect recovery. In centrifuges however, all that matters is the residence time of the particle in the centrifuge. Diluting the feed means it will just take longer to process the same amount of material for the same residence time in the centrifuge. This will just end up increasing your operating costs. However none of the equations for centrifuges are a function of the feed concentration, so this additional step would just be a waste of money. So the numerical effect is zero change in the recovery.
4. The new Σ at a higher operating speed would be $5256 \times \dfrac{6200^2}{5000^2} = 8081$ m^2.

The expected flow through the reactor to capture particles of 2μm is given by the cut-size throughput:

$$\begin{aligned} Q_{\text{cut}} &= v_{\text{TSV}}\Sigma \\ &= \left(\frac{(\rho_p - \rho_f)\, g D_{p,\text{cut}}^2}{18\mu_f}\right)\cdot\Sigma \\ &= \left(\frac{(1250-800)\,(9.81)(2\times 10^{-6})^2}{18(0.3)}\right)\cdot(8081) \\ &= 2.64\times 10^{-5}\ \text{m}^3.\text{s}^{-1} \\ &\equiv 2.28\ \text{m}^3.\text{day}^{-1} \end{aligned}$$

So it will take about $\frac{10}{2.28} = 4.4$ days to process this material.

Grading for this question: 2 + 2 + 4 + 7

Question 4 [15]

You are evaluating a pilot-scale plate and frame filter press to filter limestone particles prior to entering a municipal discharge. The plate and frame press has plates which are 400mm by 400 mm (and note that the active area is on both sides of each plate; see photos online, or in the course notes). The intention of these tests is to find the various resistances, so you can purchase a large scale unit.

The data you collect on the rental unit are posted as a spreadsheet; calculate the two resistances.

Update: there are 5 plates in the plate-and-frame pilot-scale press.

Solution

If we use the problem-solving strategy of "Define, Explore, *etc*", then the first step is to recognize that our goal is to find the resistances so that we can scale to a larger-scale unit.

There are two resistances of interest: the medium, R_{m} and the cake, R_{c}. We must also recognize that the cake resistance is a function of how long (how much filtrate, V) has passed through, as the cake builds up over time.

Also part of the define stage is to ask what we know and don't know yet. Here we see we are dealing with limestone (density can be found on various resources as about 2.6 kg.m^{-3}). We also can infer that we are dealing with a water-based system, since this is for municipal discharge, and the only material municipalities deal with in this way is water. We will assume that it is ambient conditions, so we can use the usual density and viscosity values. Another known value is the area, $A = 0.4\times 0.4\times 2\times 5 = 1.6$ m^2 .

In the "*Explore*" step we recognize that we are having to deal with the filtration equation. Now, the equation can be applied at constant pressure, and we see in the test results that the system achieves constant pressure shortly after starting up. So as we mentioned then in class, we disregard the data that is not constant pressure.

The filtration equation at constant pressure is $t = BV + \dfrac{K_p}{2}V^2$, or if we divide both sides by V, then $\dfrac{t}{V} = B + \dfrac{K_p}{2}V$. which is a linear equation with intercept $B = \dfrac{\mu R_{\text{m}}}{A(-\Delta P_{\text{total}})}$ and slope $= \dfrac{K_p}{2} = \dfrac{\mu C_S \alpha}{2A^2(-\Delta P_{\text{total}})}$.

If we examine the cake resistance term, $R_{\text{c}}(t) = \dfrac{C_S\alpha V(t)}{A}$, then note the relationship in the slope term, where slope $= \dfrac{K_p}{2} = \dfrac{\mu}{2A(-\Delta P_{\text{total}})}\cdot\dfrac{R_{\text{c}}(t)}{V(t)}$, or rearranged, it gives $R_{\text{c}}(t) = (\text{slope})\cdot\dfrac{2A(-\Delta P_{\text{total}})}{\mu}\cdot V(t)$

So our "*Plan*" is to calculate the slope $\left(\frac{K_p}{2}\right)$ and intercept (B) when plotting V on the x-axis, and $\frac{t}{V}$ on the y-axis. Then use these to calculate the resistances.

Do: the slope and intercept are shown in the online spreadsheet. This gives $B = 6.35$ s.L^{-1} $= 6350$ s.m^{-3}, so $R_{\text{m}} = \frac{BA(-\Delta P_{\text{total}})}{\mu} = \frac{(6350)(1.6)(150000)}{0.001} = 1.52 \times 10^{12}$ m^{-1}.

The slope is 1.96×10^{-2} s.L^{-2} $= 19600$ s.m^{-6}. When we substitute into the above equation we derived for the time-varying $R_c(t) = (\text{slope}) \cdot \frac{2A(-\Delta P_{\text{total}})}{\mu} \cdot V(t)$ we get values that range from 2.3×10^{12} m^{-1} to 5.1×10^{12} m^{-1}. These values are larger than the medium resistance, which is a "*Check*" you should do, as that is expected.

Question 5 [20]

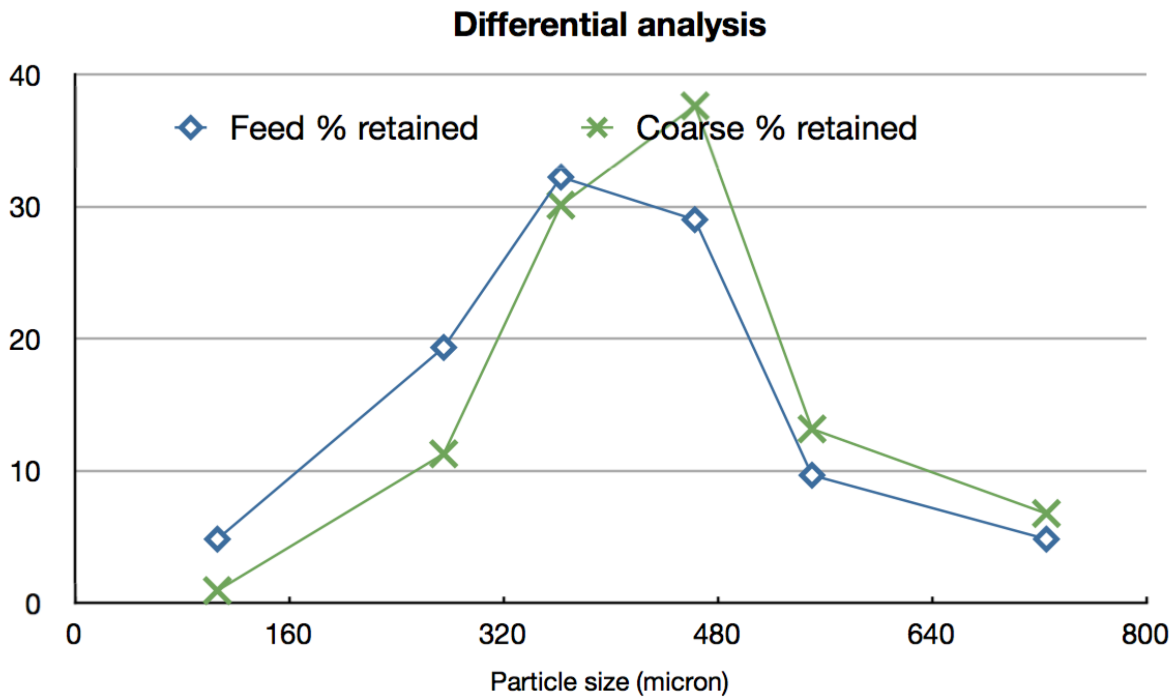
A company is running a cyclone to remove valuable dust particulates from the air stream. The solids flow rate is 180 kg solids per hour, and so far they can recover most of the solids leaving in the underflow: 127 kg solids per hour. These two streams were sampled for a period of time and using screens, the following size analyses were performed:

Mesh number	Mass retained from feed [g]	Mass retained from coarse stream [g]
20	0	0
30	30	36
35	60	70
40	180	200
50	200	160
60	120	60
Pan	30	5

1. Plot the differential analysis curve for both streams, superimposed.
2. Calculate the total efficiency of the cyclone.
3. Calculate the cut size for this cyclone.
4. Describe one way you could **quantify** the cut's sharpness (interpret what the "sharpness" when describing cyclone performance).

Solution

1. A differential analysis of the feed and coarse stream are shown below, with the numeric values shown in part 2.



Note how the coarse stream is a shifted version of the feed stream, but with higher percentages retained in the larger particle sizes, and smaller percentages for the smaller particles; as expected.

2. The total efficiency of this cyclone is $\dfrac{M_{\text{coarse}}}{M_{\text{feed}}} = \dfrac{127}{180} \approx 71\%$.

3. The cut size is calculated from the grade efficiency curve:

Mesh	Aperture	Avg aperture	Feed frac	Coarse frac	Feed flow	Coarse flow	$G(x)$
[-]	[micron]	[micron]	[%]	[%]	[kg/hr]	[kg/hr]	[fraction]
20	850	850	0.0	0.0	0.0	0.0	1.000
30	600	725	4.8	6.8	8.7	8.6	0.989
35	500	550	9.7	13.2	17.4	16.7	0.961
40	425	463	29.0	37.7	52.3	47.8	0.915
50	300	362	32.3	30.1	58.1	38.3	0.659
60	250	275	19.4	11.3	34.8	14.4	0.412
Pan	~ 212	231	4.8	0.9	8.7	1.2	0.137

The cut size is the size fraction that is at $G(x) = 0.5$, which is around 306 μm in this case. **Note**: the grade efficiency curve is calculated around the average size of the sieve above and below (as described in class).

4. One way is to estimate the slope around the $G(x) = 0.5 = 50\%$ point: the steeper the slope, the greater the sharpness. Some people use the ratio of 80% to 20% cuts, others the ratio of the 75% to 25% cuts. This ratio can be compared on different cyclones.

END