Separation Processes

ChE 4M3

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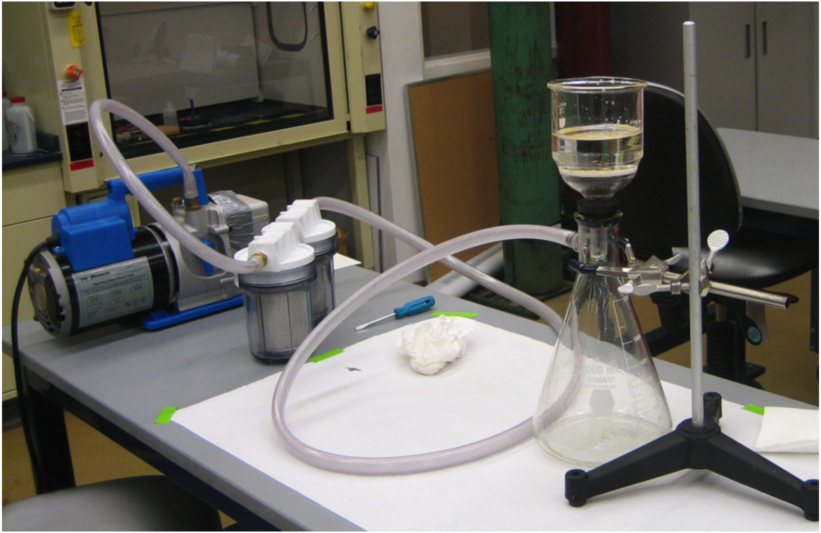
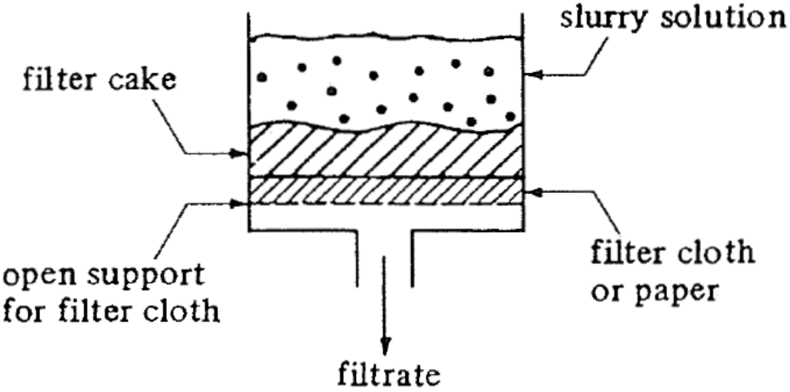
Filtration



Filtration section

Filtration: a pressure difference that causes separation of solids from slurry by means of a porous medium (e.g. filter paper or cloth), which retains the solids and allows the filtrate to pass



[Geankoplis, p 905]

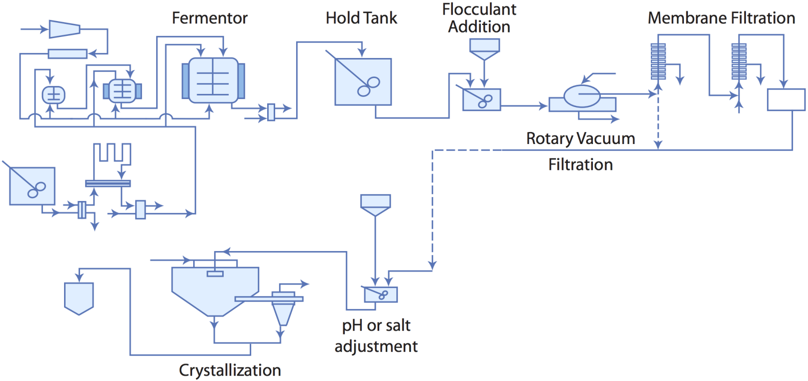
References on filtration

- Geankoplis, "Transport Processes and Separation Process Principles", 4th edition, chapter 14.
- Perry's Chemical Engineers' Handbook, 8th edition, chapter 18.
- Seader, Henley and Roper, "Separation Process Principles", 3rd edition, chapter 19.
- Uhlmann's Encyclopedia, "Filtration 1. Fundamentals",

  DOI:10.1002/14356007.b02_10.pub3

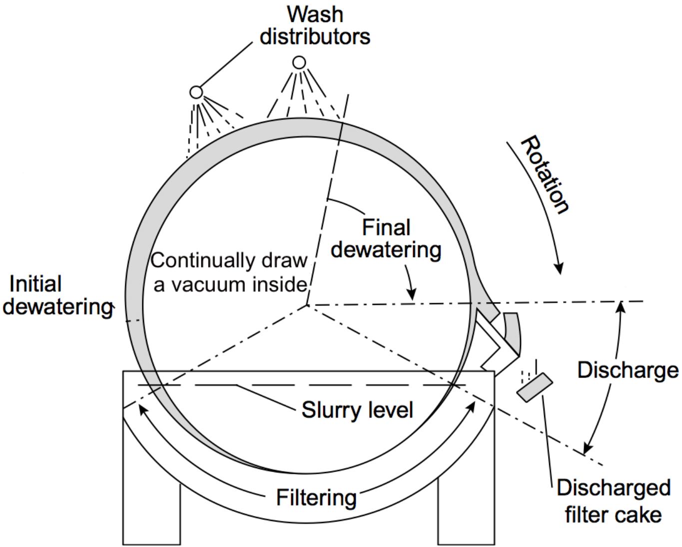
Why filtration?

Example: alkaline protease, used as an additive in laundry detergent



[MIT OCW, Course 10-445, Separation Processes for Biochemical Products, 2005]

Commercial units: rotary drum filter



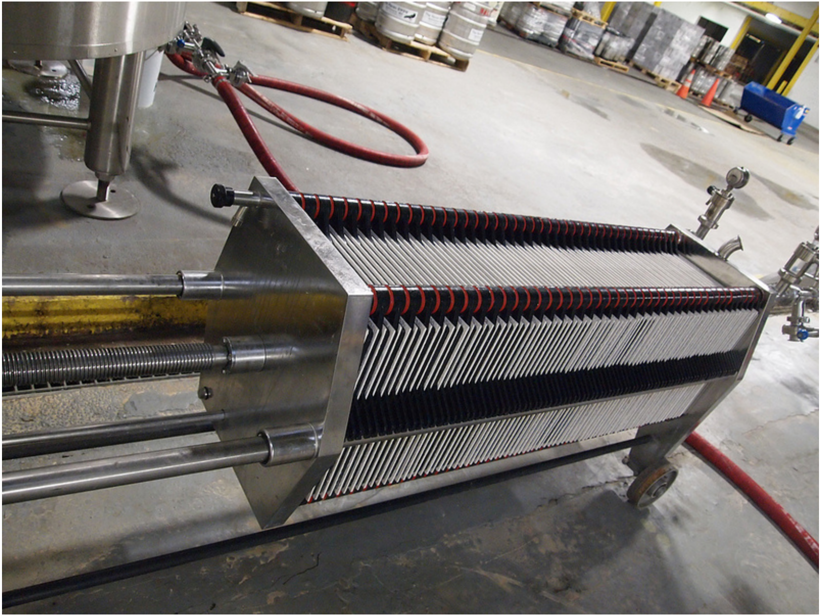
[Seader, modified from fig 19-13] [YouTube video]

Commercial units: plate and frame

Commercial units: plate and frame

[Also see YouTube video]

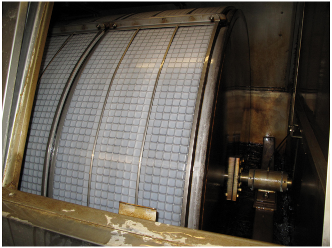
Commercial units: plate and frame (beer clarification)



Questions to discuss

1. What characteristics of a filtration system will you use to **judge** the unit's performance?

2. What factors can be used to **adjust** the units's performance?



Example: a rotary drum filter

Poiseuille's law

Recall from your fluid flow course that **laminar flow** in a pipe (considering no resistance):

$$\frac{-\Delta P}{L_c} = \frac{32\ \mu\ v}{D^2}$$

$-\Delta P$	$=$	pressure drop from start (high P) to end of tube	[Pa]
L_c	$=$	length being considered	[m]
μ	$=$	fluid viscosity	[Pa.s]
v	$=$	fluid's velocity in the pipe	[m.s^{-1}]
D	$=$	pipe diameter	[m]

Carmen-Kozeny equation through a bed of solids (cake)

$\dfrac{-\Delta P}{L_c} = \dfrac{32\ \mu\ v}{D^2}$ from which we derive the Carmen-Kozeny equation:

$$\frac{-\Delta P_c}{L_c} = k_1 \cdot \mu \cdot \frac{v}{\epsilon} \cdot \left(\frac{(1-\epsilon)S_0}{\epsilon} \right)^2$$

$-\Delta P_c$	$=$	pressure drop through the cake	[Pa]
k_1	$=$	4.17, a constant	$[-]$
ϵ	$=$	void fraction, or porosity typical values?	$[-]$
S_0	$=$	specific area per unit volume	$[\text{m}^2.\text{m}^{-3} = \text{m}^{-1}]$

- S_0 = specific surface area per unit volume is a property of the solids
- Prove in the next assignment, for spheres, $S_0 = \dfrac{6}{d} = f(d)$

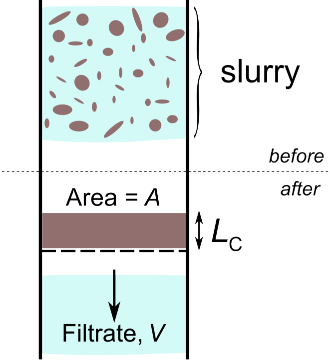
Solids balance

Mass of solids in the filter cake =
$A\ L_c\,(1-\epsilon)\,\rho_p$

Mass of fluid trapped in the filter cake =
$A\ L_c\ \epsilon\ \rho_f \approx$ small

Mass of fluid in the filtrate $= V\rho_f$

What key assumptions are being made here?



ρ_p	=	solid particle density	[kg.m^{-3}]
ρ_f	=	fluid density	[kg.m^{-3}]
V	=	volume of filtrate collected	[m^3]
A	=	cross sectional area for filtration	[m^2]

Then define slurry concentration:

$$C_S = \frac{\text{mass of dry solids}}{\text{volume of liquid in slurry}} \approx \frac{\text{mass of dry solids}}{\text{volume of filtrate}} = \frac{AL_c\,(1-\epsilon)\,\rho_p}{V}$$

Exercise

1. Calculate the mass of solids in the cake for the case when
   - $\rho_p = 3000\text{kg.m}^{-3}$
   - $A = 8\text{m}^2$
   - $L_c = 10\text{cm}$
2. Also calculate the mass of water in the cake.

Deriving the flow through the filter

Our standard equation for fluid flow:

$$\frac{1}{A} \cdot \frac{dV}{dt} = v = \frac{Q}{A}$$

for a given velocity v, and volumetric feed flow rate, Q.

But, from the Carmen-Kozeny equation:

$$\begin{aligned}
\frac{-\Delta P_c}{L_c} &= \frac{k_1 \mu v \left(1-\epsilon\right)^2 S_0^2}{\epsilon^3} \\
v &= \frac{\left(-\Delta P_c\right)\left(\epsilon\right)^3}{(\mu)(k_1)(L_c)(1-\epsilon)^2(S_0^2)}
\end{aligned}$$

from our definition for C_S we can solve for L_c

$$\begin{aligned}
L_c &= \frac{C_S V}{A\left(1-\epsilon\right)\rho_p} \\
\frac{1}{A} \cdot \frac{dV}{dt} = v &= \frac{\left(-\Delta P_c\right)(A)\cancel{(1-\epsilon)}(\epsilon)^3(\rho_p)}{(\mu)(C_S)(V)\cancel{(1-\epsilon)}(k_1)\left(1-\epsilon\right)\left(S_0^2\right)} = \frac{-\Delta P_c}{\mu C_S V \ \alpha \ /A}
\end{aligned}$$

The general filtration equation

$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_c}{\mu C_S V \alpha / A}$$

$$\begin{aligned} C_S &= \text{slurry concentration} \quad [(\text{kg dry solids})/(\text{m}^3 \text{ filtrate})] \\ \alpha &= \text{specific cake resistance} \quad [\text{m.kg}^{-1}] \end{aligned}$$

All aspects of engineering obey this general law:

$$J = \text{flux} = \frac{\text{transfer rate}}{\text{transfer area}} = \frac{\text{driving force}}{\text{resistance}}$$

including the filtration equation:

$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_c}{\mu C_S V \alpha / A}$$

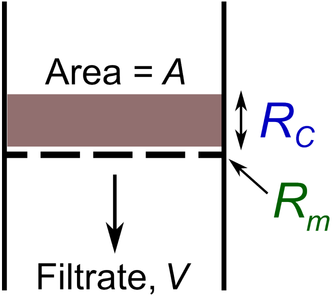
$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_c}{\mu R_c}$$

$$R_c = \text{resistance due to the cake} = \frac{C_S V \alpha}{A} \quad [\text{m}^{-1}]$$

Resistance due to the filter medium

In a similar way, we can define the filter medium's resistance:

$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_m}{\mu R_m}$$



$-\Delta P_m$	=	pressure drop across the medium	[Pa]
R_m	=	resistance due to the filter medium	[m^{-1}]

Notes:

- From a practical standpoint, R_m is empirical for the given filter
- We wrap up all other minor resistances into R_m also (e.g. pipe flow into/out of filter)
- The flux through the filter cake is exactly the same as through the medium
- After filtration gets started, we very often have $R_m \ll R_c$

Bringing it all together

As with resistances in series (you learned in Physics I), we have:

$$\frac{1}{A} \cdot \frac{dV}{dt} = \left(\frac{-\Delta P_m}{\mu R_m} \text{ add with } \frac{-\Delta P_c}{\mu R_c} \right) = \frac{-\Delta P_{\text{tot}}}{\mu \left(R_m + R_c \right)}$$

this is called the general filtration equation.

$$\begin{array}{rcll} R_c & = & \text{resistance due to the cake} & [\text{m}^{-1}] \\ R_m & = & \text{resistance due to the medium} & [\text{m}^{-1}] \\ -\Delta P_{\text{tot}} & = & \text{total pressure drop} = -(\Delta P_c + P_m) & [\text{Pa}] \end{array}$$

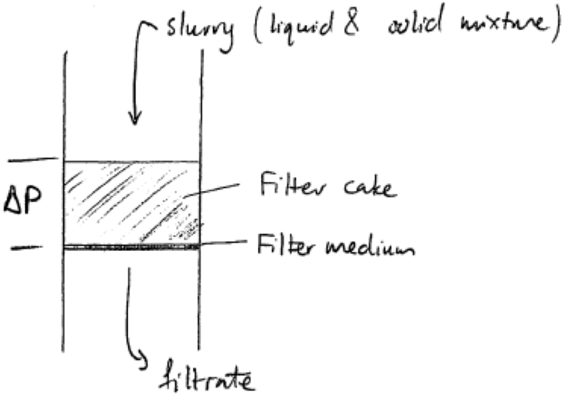
Questions

How would you use this equation?

$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_{\text{tot}}}{\mu \left(R_m + R_c \right)}$$

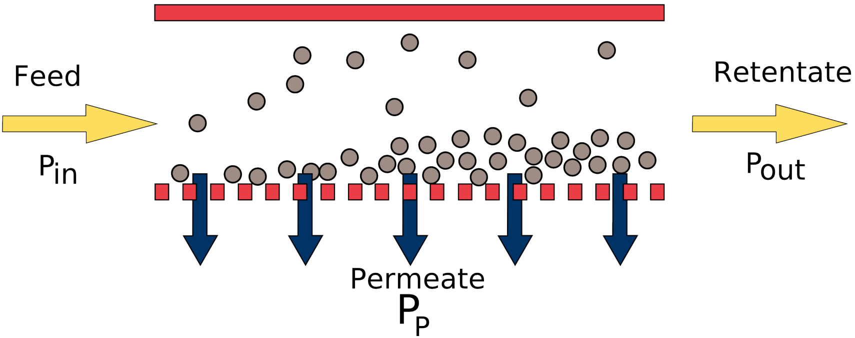
1. to determine the medium resistance?
2. to determine the cake resistance?
3. to find the utility cost of operating the filter?
4. to predict the flow for a given filter when your boss wants a higher throughput?

Cake filtration



- most widely applied
- functions only when particles have been trapped on the medium, forming a bed/cake
- the cake is the filtering element, not the medium
- medium's pores are larger than particles often
- e.g. filter press, rotary vacuum drum filter

Cross-Flow Filtration (TFF)



- TFF = tangential flow filtration (parallel to medium)
- used with gel-like or compressible substances
- pores are smaller than particles
- used when absolute exclusion is essential
- inlet flow of suspension provides shear to limit cake build-up
- reduces cake resistance, R_c

Basic filtration equation recap

$$\frac{1}{A} \cdot \frac{dV}{dt} = \frac{-\Delta P_{\text{tot}}}{\mu\,(R_m + C_S\,V\,\alpha\,/A)} = \frac{-\Delta P_{\text{tot}}}{\mu\,(R_m + R_c)}$$

$$\begin{aligned}
V &= \text{volume of filtrate collected} && [\text{m}^3] \\
A &= \text{cross sectional area for filtration} && [\text{m}^2] \\
-\Delta P_{\text{tot}} &= \text{total pressure drop} = -(\Delta P_c + P_m) && [\text{Pa}] \\
\mu &= \text{fluid viscosity} && [\text{Pa.s}] \\
C_S &= \text{slurry concentration} && \left[{\scriptstyle (\text{kg dry solids})/(\text{m}^3\ \text{filtrate})}\right] \\
R_m &= \text{resistance due to the medium} && [\text{m}^{-1}] \\
R_c &= \text{resistance due to the cake} && [\text{m}^{-1}] \\
\alpha &= \text{specific cake resistance} && [\text{m.kg}^{-1}] \\
\alpha &= \frac{(k_1)\,(1-\epsilon)\,(S_0^2)}{(\epsilon)^3(\rho_p)}
\end{aligned}$$

- which entries in the equation are a function of t?
- which entries in the equation are a function of V?

How do we determine cake resistance α?

Recall:

$$\alpha = \frac{(k_1)(1-\epsilon)(S_0^2)}{(\epsilon)^3(\rho_p)}$$

- Measuring S_0 is difficult for irregular solids
- ϵ changes depending on many factors (surface chemistry, upstream processing)
- Is ϵ constant over time? What does it change with?
- α will also change as a function of ΔP
- So we let $\alpha = \alpha_0 (-\Delta P)^f$

Constant pressure (batch) filtration

$$\frac{1}{A}\cdot\frac{dV}{dt} = \frac{-\Delta P_{\text{tot}}}{\mu\,(R_m + V C_S\,\alpha\,/A)} = \frac{-\Delta P_{\text{tot}}}{\mu\,(R_m + R_c)}$$

Invert both sides, separate, divide by A

$$A\cdot\frac{dt}{dV} = \frac{\mu\,(R_m + V C_S\,\alpha\,/A)}{-\Delta P_{\text{tot}}}$$

$$\frac{dt}{dV} = \frac{\mu}{A\,(-\Delta P_{\text{tot}})}R_m + \frac{\mu C_S \alpha}{A^2\,(-\Delta P_{\text{tot}})}V$$

$$\frac{dt}{dV} = B + K_p V$$

$$\int_0^t dt = \int_0^V (B + K_p V)\,dV$$

$$t = BV + \frac{K_p V^2}{2}$$

$$B = \frac{\mu}{A\,(-\Delta P_{\text{tot}})}R_m \quad [\text{s.m}^{-3}] \qquad \text{and} \qquad K_p = \frac{\mu C_S \alpha}{A^2\,(-\Delta P_{\text{tot}})} \quad [\text{s.m}^{-6}]$$

Constant pressure (batch) filtration

$$t = BV + \frac{K_p V^2}{2}$$

Practical matters:

- how do we get constant $-\Delta P$?
- which type of equipment has a constant pressure drop?
- what is the expected relationship between t and V?
- Plot it for a given batch of slurry.
- If $-\Delta P$ is constant, then ϵ is likely constant, and so is α

Example

A water-based slurry of mineral is being filtered under vacuum, with a controlled pressure drop of 38 kPa, through a filter paper of 0.07 m^2. The slurry is at 24 kg solids per m^3 fluid. Use $\mu = 8.9 \times 10^{-4}$ Pa.s and the following data:

V [L]	t [s]	t/V [s/L]
0.5	19	37
1	38	38
2	95	48
3	178	60
4	280	70

1. plot a rough sketch of $x = V$ against $y = t/V$
2. read off the intercept and slope
3. calculate medium resistance, R_m [*Ans*: $8.11 \times 10^{10} \mathrm{m}^{-1}$]
4. calculate specific cake resistance, α [*Ans*: $1.87 \times 10^{11} \mathrm{m.kg}^{-1}$]
5. cake resistance R_c at $t = 280$ s [*Ans*: $2.56 \times 10^{11} \mathrm{m}^{-1}$]

Pressure dependence of α

Recall: $\alpha = \alpha_0 \left(-\Delta P\right)^f$

- We can find tables of α_0 and f for various solids
- But they almost will never match our situation

Finding the α value as a function of pressure drop:

1. Repeat the example above at several different $-\Delta P$ levels
2. Calculate α at each $-\Delta P$ (see example above)
3. Plot $x = (-\Delta P)$ against $y = \alpha$ (expected shape?)
4. Take logs on both sides of equation:

$$\ln\left(\alpha\right) = \ln\left(\alpha_0\right) + f \ln\left(-\Delta P\right)$$

Note: if $f = 0$ the cake is called an "incompressible cake"

Constant rate filtration

At a **constant rate**, i.e. $\frac{dV}{dt} = \text{constant}$

$$\frac{1}{A}\frac{dV}{dt} = \frac{1}{A}\frac{V}{t} = \frac{Q}{A} = \frac{-\Delta P}{\mu\left(R_m + C_S V \alpha / A\right)}$$

We can solve the equations for $-\Delta P$

$$-\Delta P = \frac{\mu R_m V}{At} + \frac{\mu C_s \alpha V^2}{t \cdot A^2}$$
$$-\Delta P = \frac{\mu R_m Q}{A} + \frac{\mu C_s \alpha Q^2}{A^2} t$$

- For many processes, the constant rate portion is fairly short
- Occurs at the start of filtration, where a plot of $x = (t)$ against $y = (-\Delta P)$ is linear
- Then we settle into constant pressure mode
- That's the part we have focused on earlier

Example continued

$$t = BV + \frac{K_p V^2}{2} \qquad B = \frac{\mu}{A\left(-\Delta P_{\mathrm{tot}}\right)} R_m \quad \text{and} \quad K_p = \frac{\mu C_S \alpha}{A^2\left(-\Delta P_{\mathrm{tot}}\right)}$$

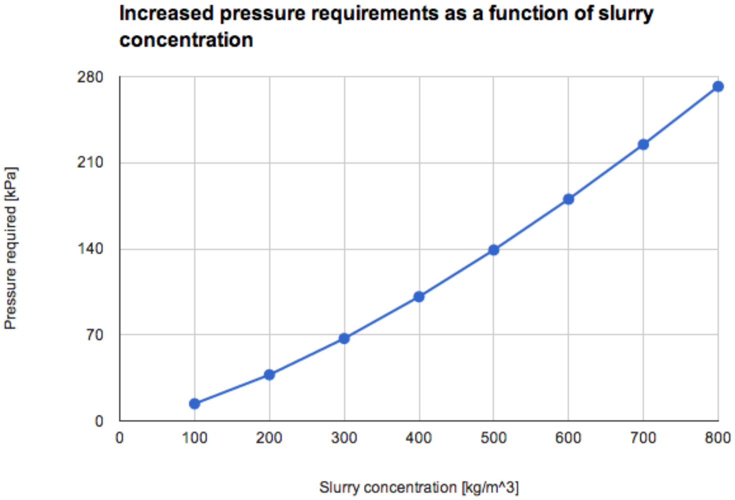
Example continued

Suppose our prior lab experiments determined that $\alpha = 4.37 \times 10^9 \left(-\Delta P\right)^{0.3}$, with $-\Delta P$ in Pa and α in SI units. Now we operate a plate and frame filter press at $-\Delta P$ of 67 kPa,

with solids slurry content of $C_s = 300 \ \dfrac{\text{kg dry solids}}{\text{m}^3 \text{ filtrate}}$. The cycle time that the filter actually operates is 45 minutes (followed by 15 minutes for cleaning). Based on a simple mass-balance, the company can calculate that 8.5 m^3 of filtrate will be produced.

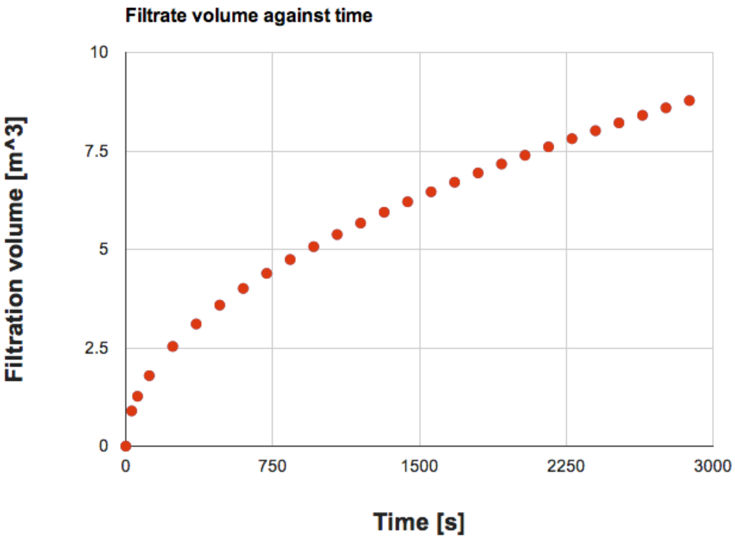
1. Calculate the area required.
2. If the slurry concentration had to double (still the same volume of filtrate), what would the required pressure drop have to be to maintain the same cycle time?
3. Create a plot of the volume of filtrate leaving the press as a function of time, t.

Pressure requirements change with concentration of slurry



- $A = 81$ m^2 stays fixed
- $t = 2700$ s stays fixed

Volume of filtrate produced against time



- $A = 81\ \text{m}^2$ stays fixed

Extend your knowledge

Research the following topics:

- Variable rate filtration: pressure and flow rate both vary along the characteristic centrifugal pump curve
- Filtering centrifuge (combine the topics from the last 2 weeks)