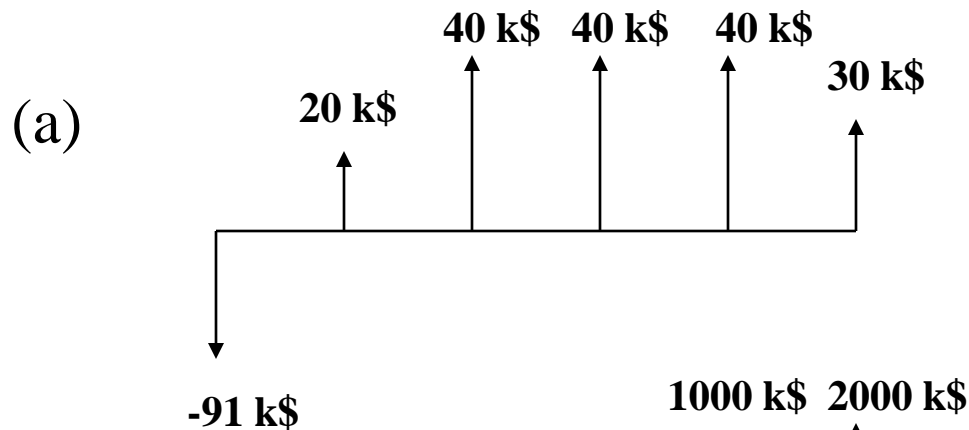
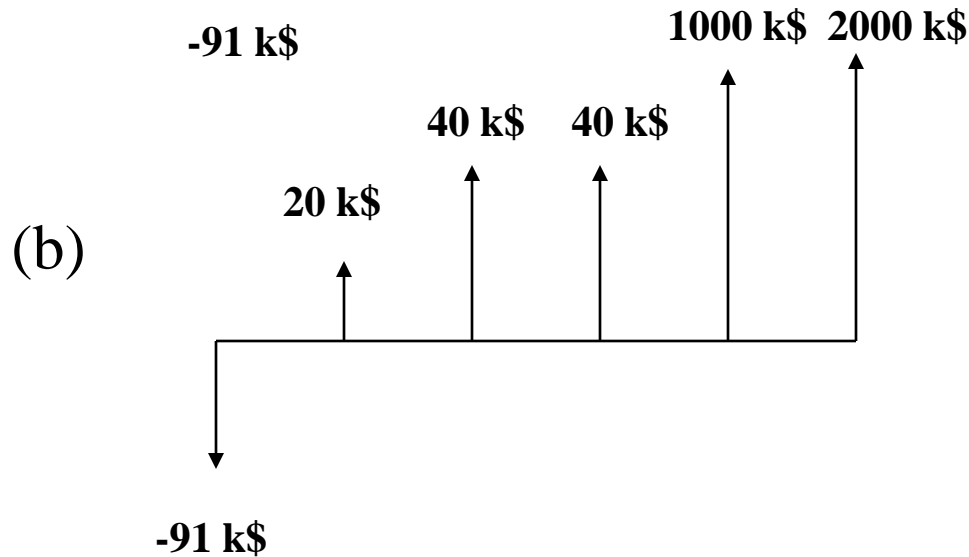
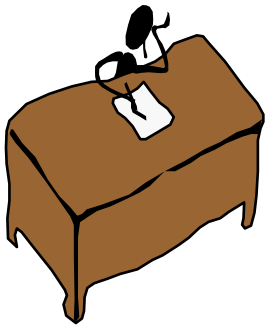


1. Payback time: Calculate the payback time for the two cash flows given below and discuss the differences in your results. (Arrows not to scale)

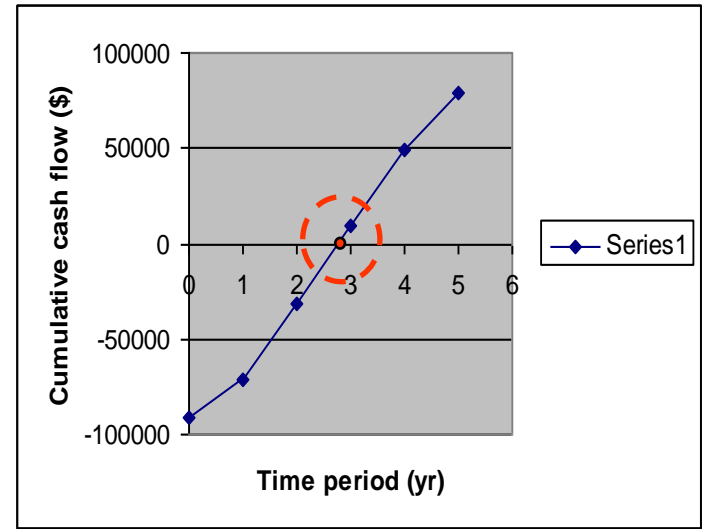
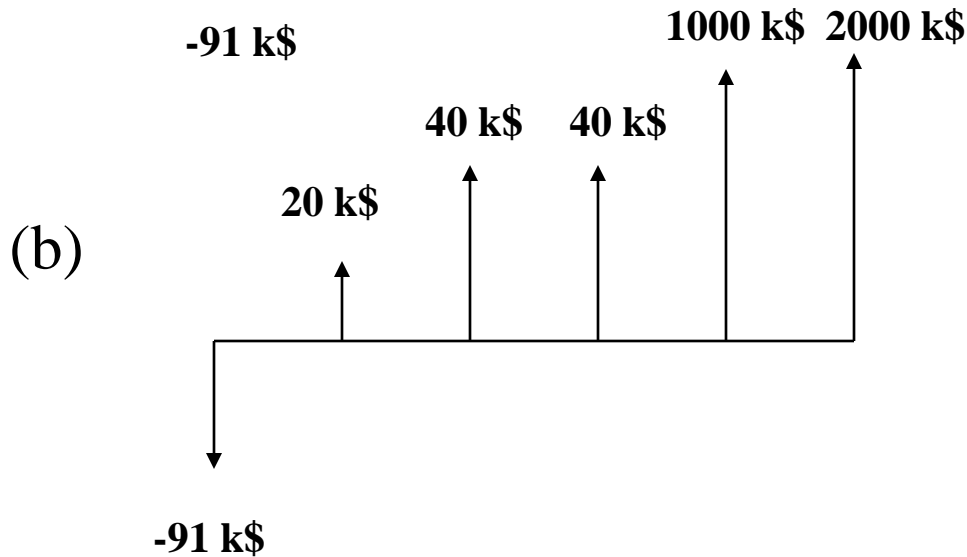
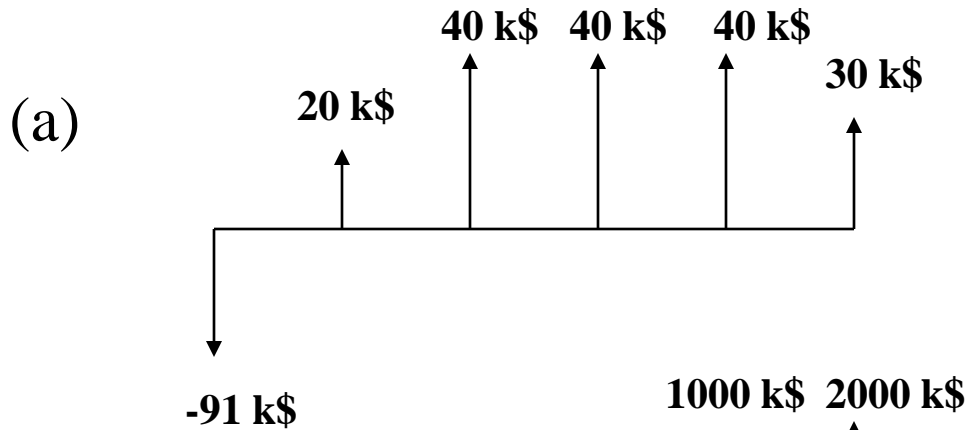


(a) This is the cash flow used in several lecture examples.





1. Payback time: Solution

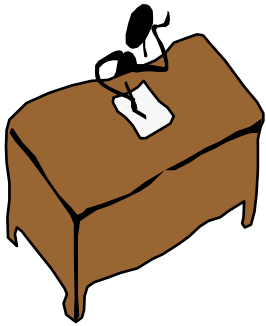


Payback time = ~2.7 years

DCFRR = 23.6 %

Payback time = ~2.7 years

DCFRR = 127 %



1. Payback time: **Solution**

(a) Payback time = ~2.7 years

DCFRR = 23.6 %

(b) Payback time = ~2.7 years

DCFRR = 127 %

Payback time considers only the cash flows up to when the cumulative cash flow first reaches zero.

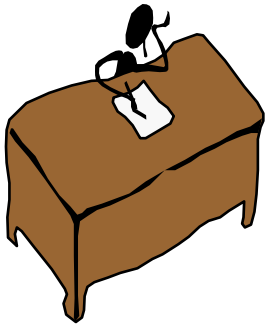
The profitability of a project depends on the time value of money and all cash flows.

In this example, very large cash flows occur in (b) after the payback time.

Therefore, (b) is much more financially attractive.

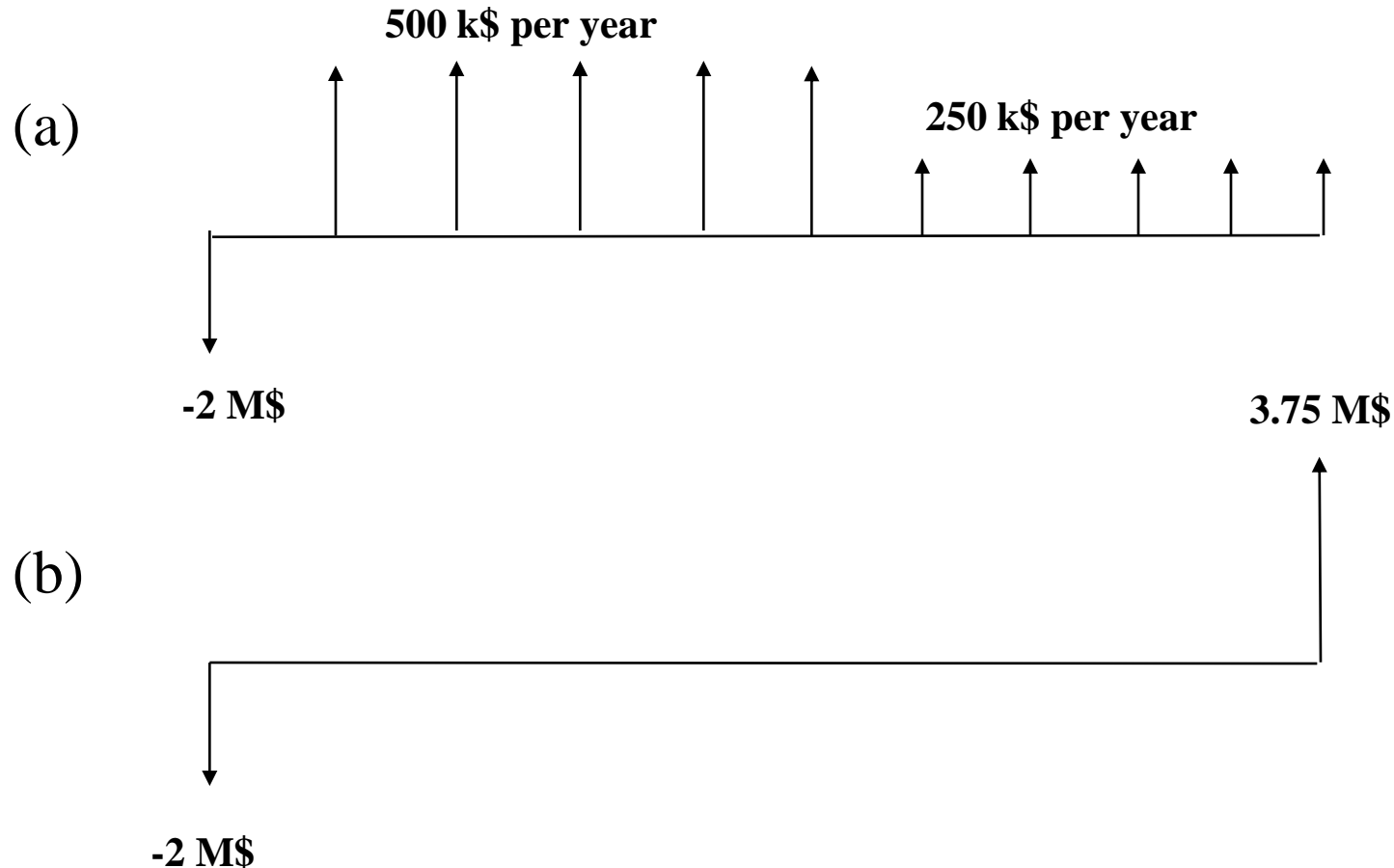
The payback time analysis gives a faulty evaluation of these projects.

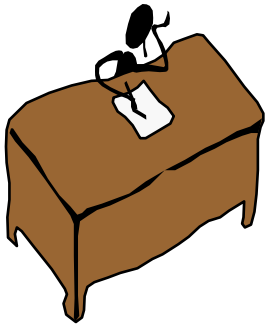
This example demonstrates a serious weakness in the payback method.



2. Return on original Investment (ROI):

Calculate the ROI for the two cash flows given below and discuss the differences in your results. (Arrows not to scale. No working capital.)





2. Return on original Investment (ROI): Solution

$$\text{ROI} = (\text{average annual profit})/(\text{fixed capital}+\text{working capital})$$

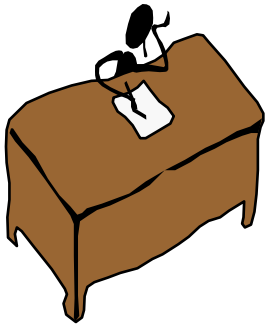
$$(a) \quad \text{ROI} = \frac{375000}{2000000} = 0.188 \text{ or } 18.8\%$$

With a MARR = 12%

$$\text{NPV}(a) = 0.31 \text{ M\$}$$

$$(b) \quad \text{ROI} = \frac{375000}{2000000} = 0.188 \text{ or } 18.8\%$$

$$\text{NPV}(b) = -0.80 \text{ M\$}$$



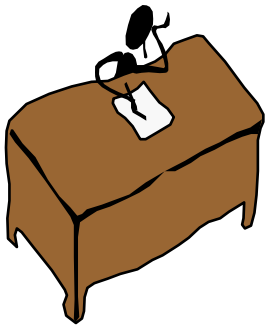
2. Return on original Investment (ROI): Solution

$$\text{ROI} = (\text{average annual profit}) / (\text{fixed capital} + \text{working capital})$$

The ROI does **not consider the time value of money**. Therefore, the two projects are found to be equivalent using ROI because they have the same average profit over the life of the project.

However, Project (a) has positive cash flows earlier in the project, and therefore, it is more financially attractive, as confirmed by their NPV's.

This example demonstrates a serious weakness in the ROI method.

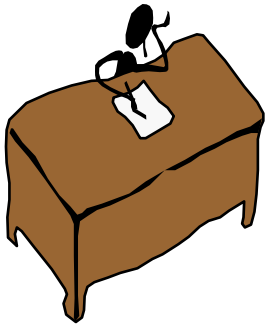


3. Net Present Value (NPV) for pump:

Calculate the *before-tax* NPV for a pump using the following data.

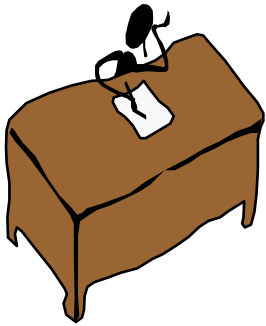
Initial installed cost = \$2500	Salvage value = \$200
Annual operating cost = \$900	Pump life = 5 years
MARR = 10%	Project life = 5 years





3. Net Present Value (NPV) for pump: Solution.

year	0	1	2	3	4	5
capital	-2500	0	0	0	0	0
Operat. cost	0	-900	-900	-900	-900	-900
Salvage	0	0	0	0	0	200
Cash flow	-2500	-900	-900	-900	-900	-700
Discount factor	1	.91	.83	.75	.68	.62
Present value	-2500	-819	-747	-675	-612	-434
NPV = Sum (present values) = \$ -5787						



4. Annualized measure of profit:

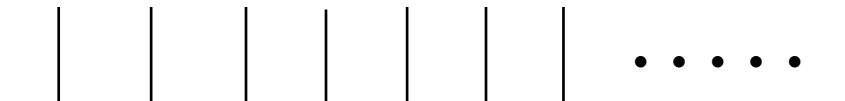
In some circumstances, people like to express the net effect of all economic factors in an annualized manner. This must account for the time value of money. Develop an annualized equivalent to NPV.

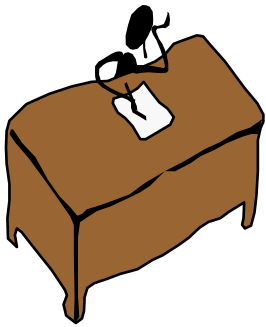
- What (equal) annual cash flow is equivalent to the NPV value at an interest rate of i ?
- What is the criterion for an attractive investment?

NPV



EAV = equivalent annual value (worth)





4. Annualized measure of profit:

Solution.

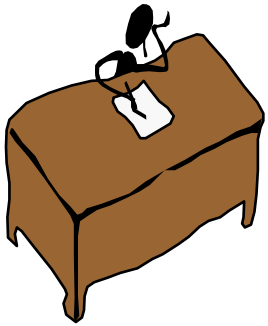
Write the expression equating the NPV and EAV. We will use “A” for EAV and “P” for NPV, as is done in many interest tables.

$$P = A \left(\frac{1}{(1+i)} \right) + A \left(\frac{1}{(1+i)^2} \right) + A \left(\frac{1}{(1+i)^3} \right) + \dots$$

This expression can be simplified to the following.

$$P = A \left(\frac{(1+i)^n - 1}{i(1+i)^n} \right) \quad \text{Factor} = (P/A, i, n)$$

These factors are available in the interest tables in books or in Excel functions.



4. Annualized measure of profit:

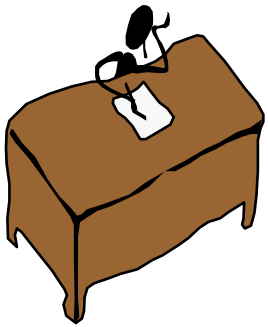
Solution.

As an example, what is the equivalent NPV for cash flows of \$1000 in years 1 to 9 when the MARR = 15%?

The answer is **not** $1000 * 9 = 9000$ because of the time value of money.

$$P = 1000 (P/A, 0.15, 9) = 1000 * 4.7716 = \$ 4772$$

For an attractive investment, the EAV must be positive when using the MARR for the compound interest rate.

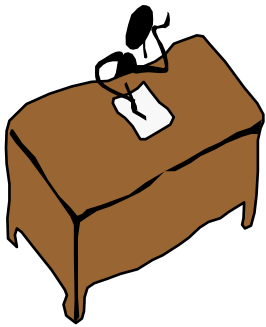


5. Comparing Alternatives, repeated

projects: In some instances, a project has an essentially infinite life. Alternatives often can be repeated without extra cost.

Problem: A process needs a pump, and the need will exist for a very long time. There are two alternative pumps available. Each requires the same energy for operation, can be installed on January 1 and operated immediately; time to place the order, deliver and install it is ~ 1 year. Pump only operates in period $n=1$ and onwards. $MARR = 15\%$. Determine the lowest cost alternative.

	Pump A	Pump B
Installed cost	\$ 9500	\$ 22000
Salvage value	\$ 0	\$ 0
Life	2 years	5 years



5. Comparing Alternatives, repeated projects: **Solution I.**

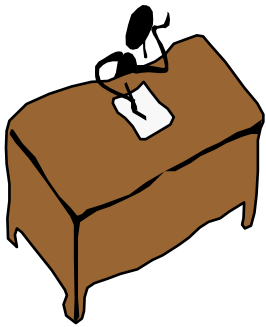
We can compare alternatives using NPV, DCFRR, and so forth when the project lives are the same. Since this project has essentially infinite life, we can use any period in which the projects have the same life. We use the **“least common multiple method”**.

Same project lives

	Project has infinite life										Least common multiple method Equivalent Annual value (worth) method											
Pump A	-9500																					
year	0	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8	9	10
CF	0	-9500	0	-9500	0	-9500	0	-9500	0	-9500	0	0	-9500	0	-9500	0	-9500	0	-9500	0	-9500	0
PV	0	-8260.87	0	-6246.4	0	-4723.18	0	-3571.4	0	-2700.49	0	0	-8260.87	0	-6246.4	0	-4723.18	0	-3571.4	0	-2700.49	0
NPV	-25502.34755											-25502.34755										
Pump B	-22000																					
year	0	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8	9	10
CF	0	-22000	0	0	0	0	-22000	0	0	0	0	0	-22000	0	0	0	-22000	0	0	0	0	0
PV	0	-19130.4	0	0	0	0	-9511.21	0	0	0	0	0	-19130.4	0	0	0	-9511.21	0	0	0	0	0
NPV	-28641.64189											-28641.64189										

-25502.34755

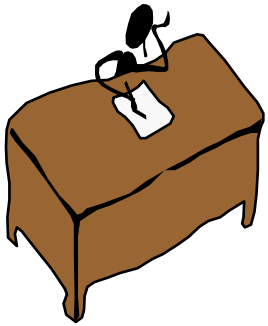
Better choice



5. Comparing Alternatives, repeated projects: **Solution II.**

The projects can be repeated without cost. Therefore, we could also compare the annualized cost for each alternative for one “cycle”. We use the “**equivalent annual worth**”.

Pump A	This is a two-year project, with 9500 invested in year one			
	Invest	-9500		
	NPV	-8260.87		
	EAV factor	0.615116	(A/P, .15, 2)	
	EAV	-5081.4		
			Better choice	
Pump B	This is a five-year project with 22000 invested in year one			
	Invest	-22000		
	NPV	-19130.4		
	EAV factor	0.298316	(A/P, .15, 5)	
	EAV	-5706.91		

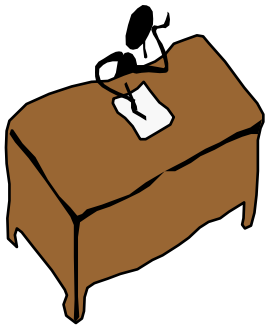


6. Comparing Alternatives, with fixed project life:

In some instances, a project has fixed life. Alternative plans must satisfy the total life.

Problem: A process needs a pump, and the need will exist for six years. There are two alternative pumps available. Each requires the same energy for operation. $MARR = 15\%$. Determine the lowest cost alternative.

	Pump A	Pump B
Installed cost	\$ 9500	\$ 22000
Salvage value	\$ 0	\$ 0
Life	2 years	6 years



6. Comparing Alternatives, with fixed project life: **Solution**

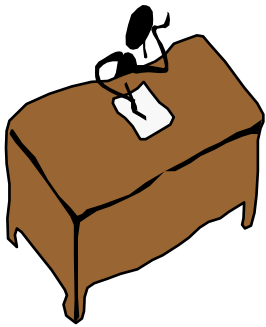
Pump A	-9500					
year	0	1	2	3	4	5
CF	-9500	0	-9500	0	-9500	0
PV	-9500	0	-7183.36	0	-5431.66	0
NPV	-22115					

Requires three pump purchases.

Pump B	-22000					
year	0	1	2	3	4	5
CF	-22000	0	0	0	0	0
PV	-22000	0	0	0	0	0
NPV	-22000					

Requires one pump purchase

Better choice



9. After-tax profitability:

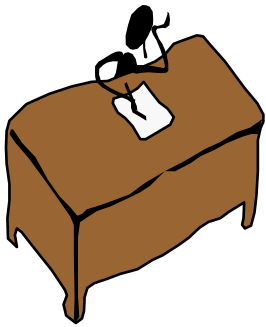
Answer question 8-10 from Peters, Timmerhaus and West.

8-10 A proposed chemical plant has the following projected revenues and operating expenses in millions of dollars

Year	Annual revenue	Annual operating expenses (excluding depreciation)
1	7.0	4.0
2	10.0	5.6
3	15.0	6.8
4	20.0	7.8
5	22.5	8.8
6	24.0	9.6
7	25.0	10.0

The fixed-capital investment for the plant is \$50 million with a working capital of \$7.5 million. Using a MACRS depreciation schedule with a class life of 5 years, determine

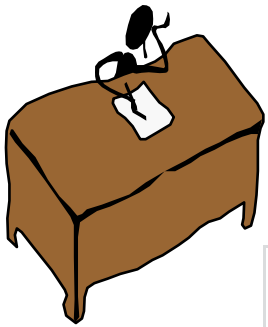
- The annual cash flows
- The net present worth, using a nominal discount rate of 15 percent



13. NPV measure of profitability

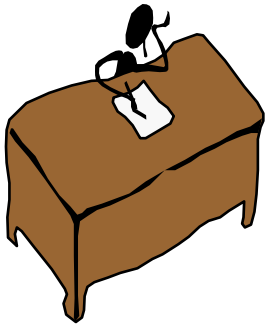
The following cash flow is given for two projects.
Determine the NPV for each and discuss the results.

Profitability of two projects						
				MARR =	0.12	
Project 1						
	year					
	0	1	2	3	4	5
cash flow	-4000	1300	1300	1300	1300	1300
Project 2						
	year					
	0	1	2	3	4	5
cash flow	-4000000	1109829	1109829	1109829	1109829	1109829



13. NPV measure of profitability

Profitability of two projects							
				MARR =	0.12		
Project 1							
	year						
		0	1	2	3	4	5
cash flow		-4000	1300	1300	1300	1300	1300
PV		-4000	1160.714	1036.352	925.3143	826.1735	737.6549
NPV		686.2091					
DCFRR		18.72%					
Project 2							
	year						
		0	1	2	3	4	5
cash flow		-4000000	1109829	1109829	1109829	1109829	1109829
PV		-4000000	990919	884749.1	789954.6	705316.6	629746.9
NPV		686.2					
DCFRR		12.01%					



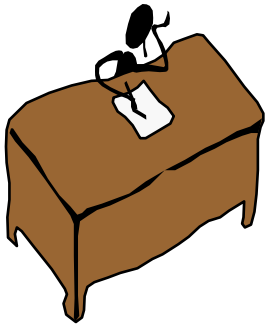
13. NPV measure of profitability

**The following cash flow is given for two projects.
Determine the NPV for each and discuss the results.**

We note that the **NPV values are the same for both projects**. However, Project 2 involves an investment of one thousand times the investment for Project 1. We also note that the **DCFRR for Project 1 is higher than Project 2**.

We have seen a number of advantages for NPV (simpler calculations for exclusive alternatives, no controversy regarding reinvestment assumption), while this example displays a shortcoming for the NPV profitability measure.

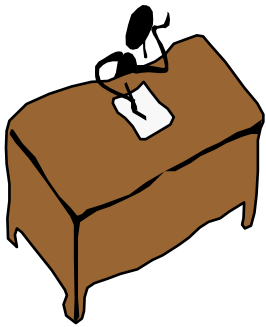
Conclusion: Use NPV and DCFRR and observe the cash flows.



14. Effect of uncertainty

Let's calculate the before-tax profitability (NPV) for the following cash flows. Also, let's evaluate the sensitivity to $\pm 20\%$ uncertainty in the initial investment in year 0 and the cash flows in years 1-5.

interest (MARR) =		12 %					
Calculate the profitability							
year		0	1	2	3	4	5
cash flow		-4000000	1109829	1109829	1109829	1109829	1109829

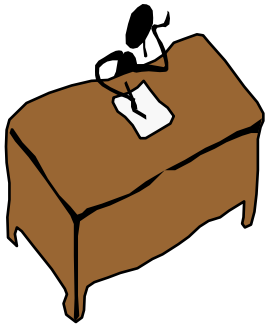


14. Effect of uncertainty

The base case solution is given in the following table. Often, students report that the **profitability = $685 \pm 685 \cdot .20$** .

Is this correct?

				uncertainty (%)			
investment =		-4000000		0	(range 0-20%)		
Annual cash flow =		1109829		0	(range 0-20%)		
interest (MARR) =		12 %					
Calculate the profitability							
year		0	1	2	3	4	5
cash flow		-4000000	1109829	1109829	1109829	1109829	1109829
PV		-4000000	990918.8	884748.9	789954.3607	705316.4	629746.8
NPV =	685.1679						
DCF =	12.01%						



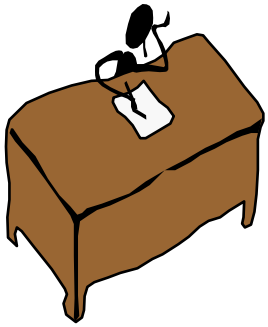
14. Effect of uncertainty

The proposed answer is not correct!

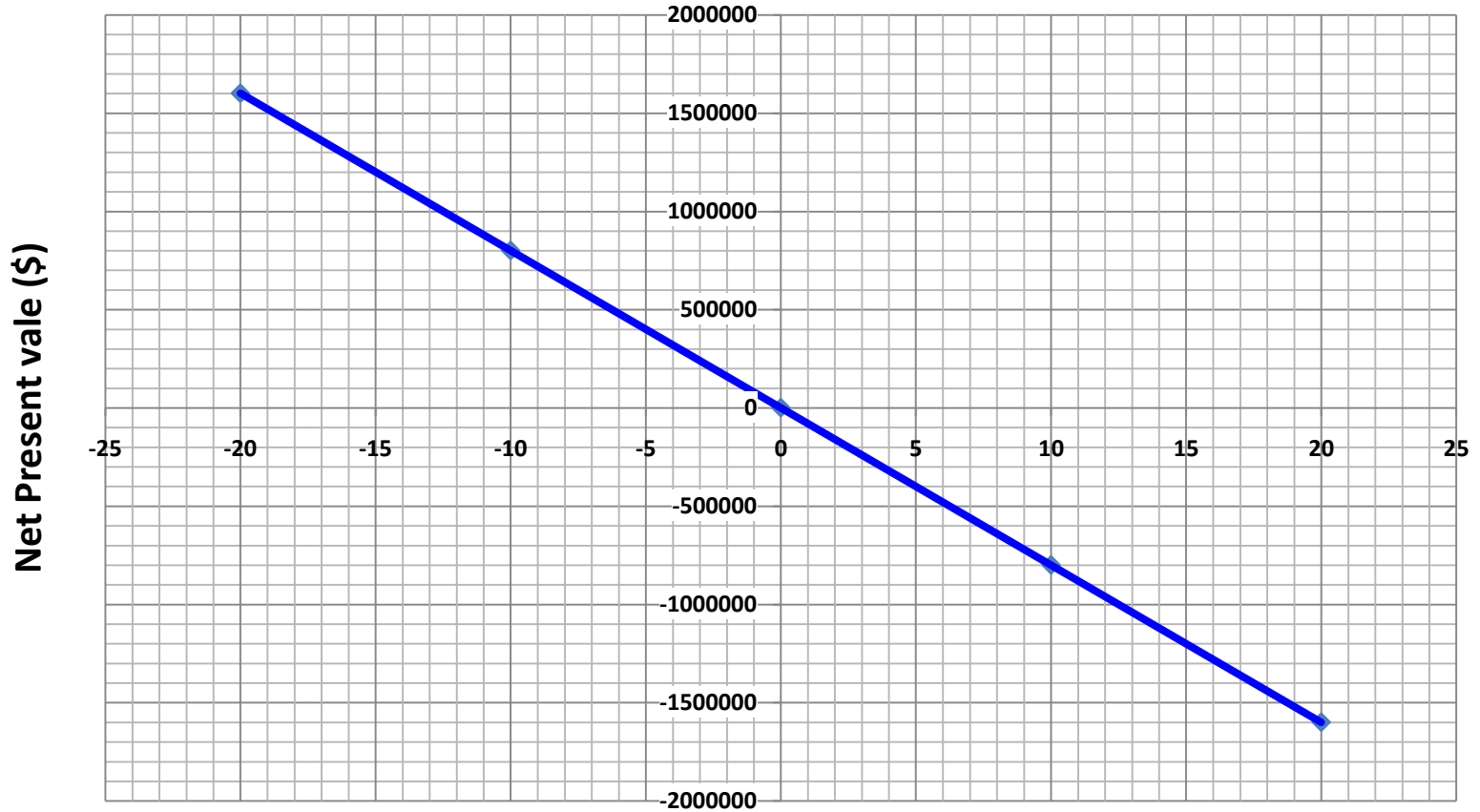
The proper method depends upon the correlation among the uncertainties in the various sources of uncertainty. Here, we will consider them independent. Thus, we set the uncertainties to their extreme magnitudes with different signs.

The result is shown in the figure on the following slide, showing values that differ from the base case by over a factor of 1000!

Lesson: When calculating the difference between large numbers, even small changes in the numbers can have a large affect on the result.



14. Effect of uncertainty



Uncertainty in cash flows (%), Opposite signs for investment and revenues; sign is for investment in plot