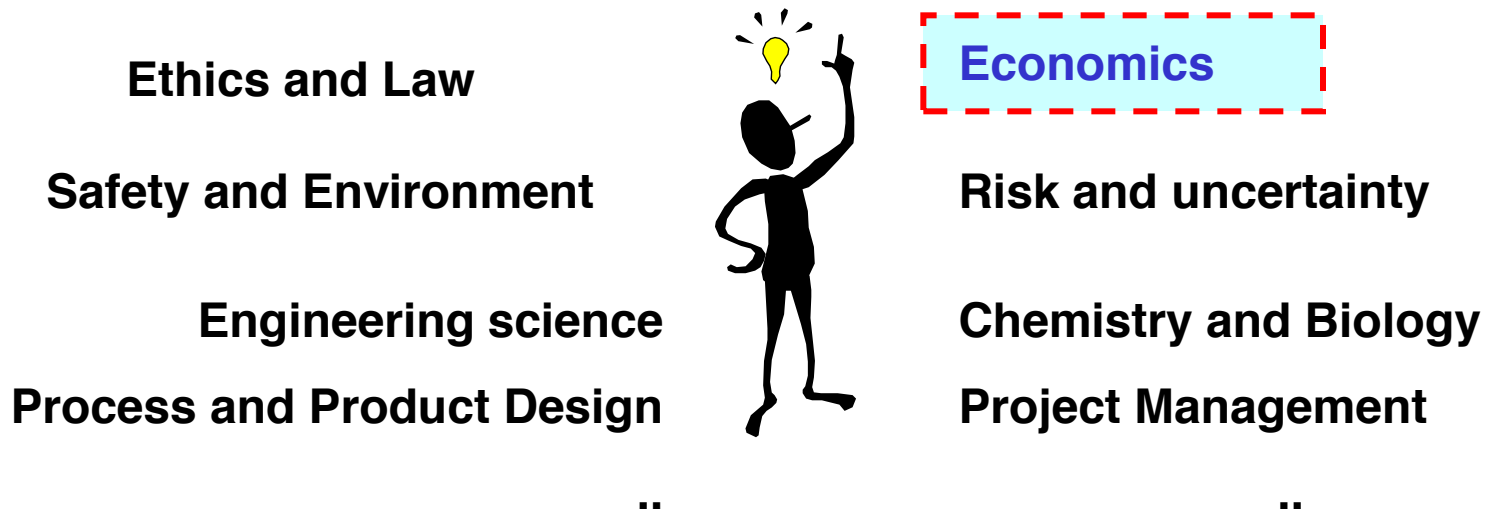


# Class notes for ChE 4N04 Engineering Economics section



**We all must be able to apply basic concepts of economics because economics plays an important role in every engineering decision.**

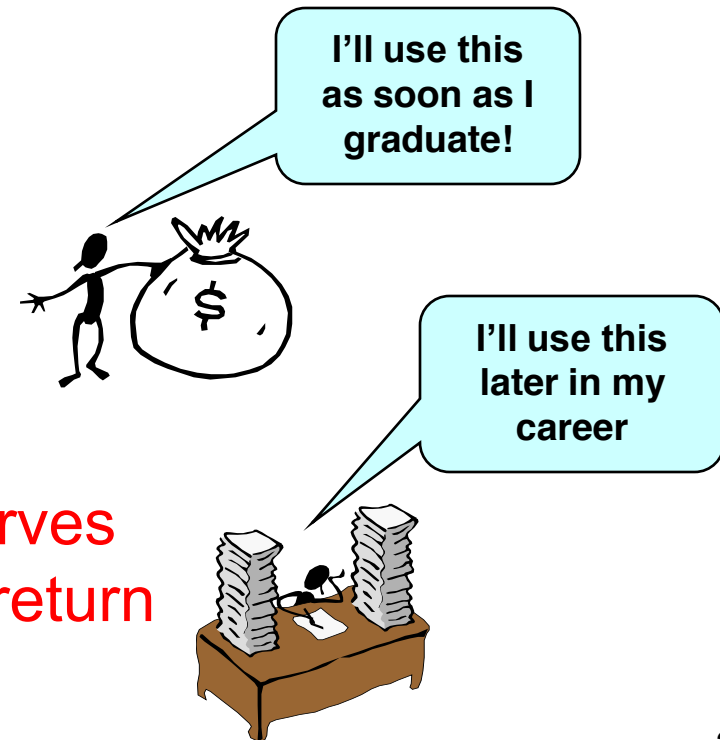
# Course principles have many applications

## Engineering Economics

- Evaluate profitability of alternative investments

## Personal Finance

- When to buy that new car!
- Determine proper level of borrowing and saving
- Calculate income taxes



## Corporate Finance

- Provide adequate cash reserves
- Determine minimum rate of return

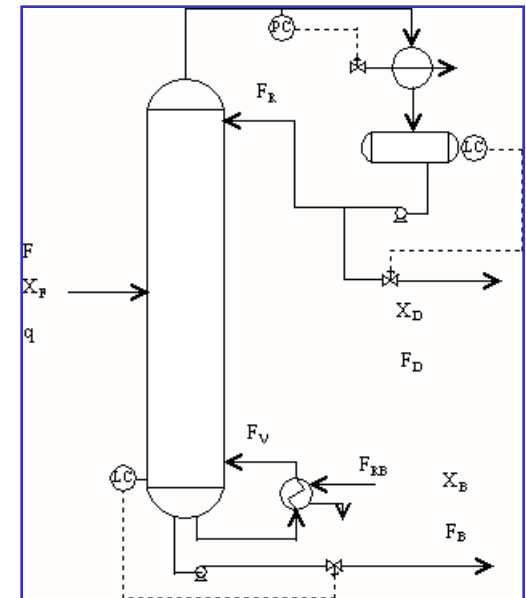
# Your first task at your new job

**Supervisor to you:** We want to increase our production rate by 35%, but the distillation tower is at its maximum capacity (liquid and vapour flows).

**Evaluate the following feasible alternatives and determine the most financially attractive.**

*After some creative brainstorming ...*

1. Build a parallel distillation tower
2. Replace trays with packing
3. Increase the number of trays
4. Contract the extra production to another company
5. Change operating conditions



What is the best choice?



# Roadmap for engineering economics topic

- **Four major topics**

- Time value of money
- Quantitative measures of profitability
- Selecting from among alternatives
- Cost estimating



**Able to evaluate potential projects and select the best**

- **Lecture exercises and thought questions**
- **Class workshop**
- **Midterm (individual)**
- **Application in the SDL Project**

# Four major topics in engineering economics

## 1. Time value of money

- How do we compare \$ at different times?

## 2. Quantitative measures of profitability

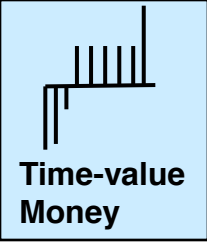
- How do we determine the “profit” or “financial attractiveness” from an investment?

## 3. Systematic comparison of alternatives

- How do we ensure that we select the “best” investment?

## 4. Estimation of costs

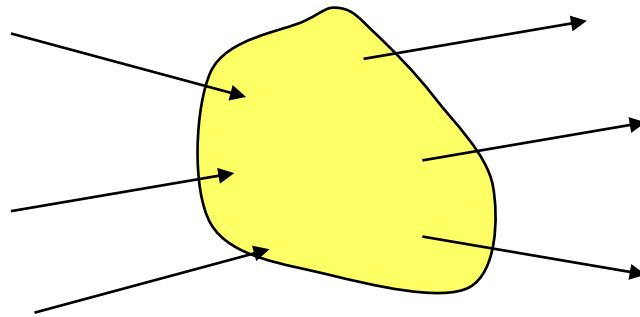
- How do we determine the costs before we buy?



# Time value of money

Let's use our modeling skills to determine a “*money balance*”

**Revenues or incomes flow into the system, e.g.**

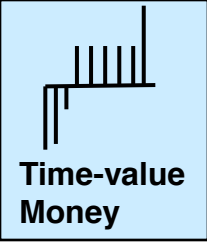


**Expenditures or costs flow out of the system, e.g.,**

- Product sales
- Equipment sales
- Licensing fees

- Feed costs
- Fuel and electricity
- Employee salaries

**Important definition:** **Cash flows** are transfers of money that cross the system boundary. The system is typically a “project”.



# Time value of money

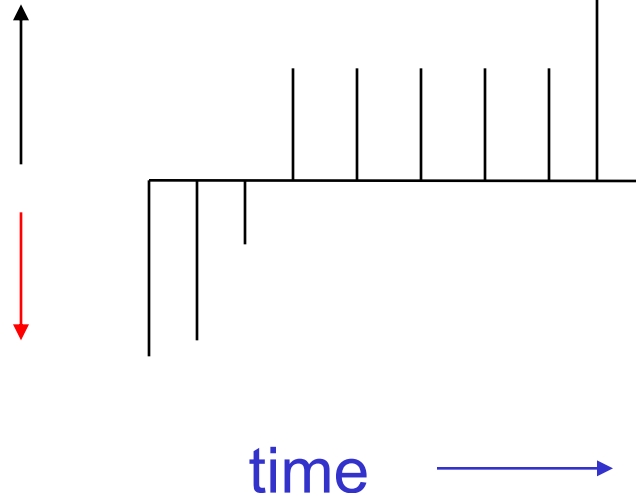
## Cash flows occur over time.

We sum the revenues and expenditures within each time period to give the net cash flow at a time. We plot these in a cash flow diagram.

## Cash flow diagram

**Positive  
cash  
flow**

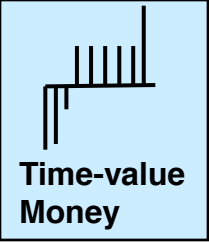
**Negative  
cash flow**



**Periods are  
numbered from 0 to  
the end of analysis.**

Period can be any time length; often one year for engineering projects

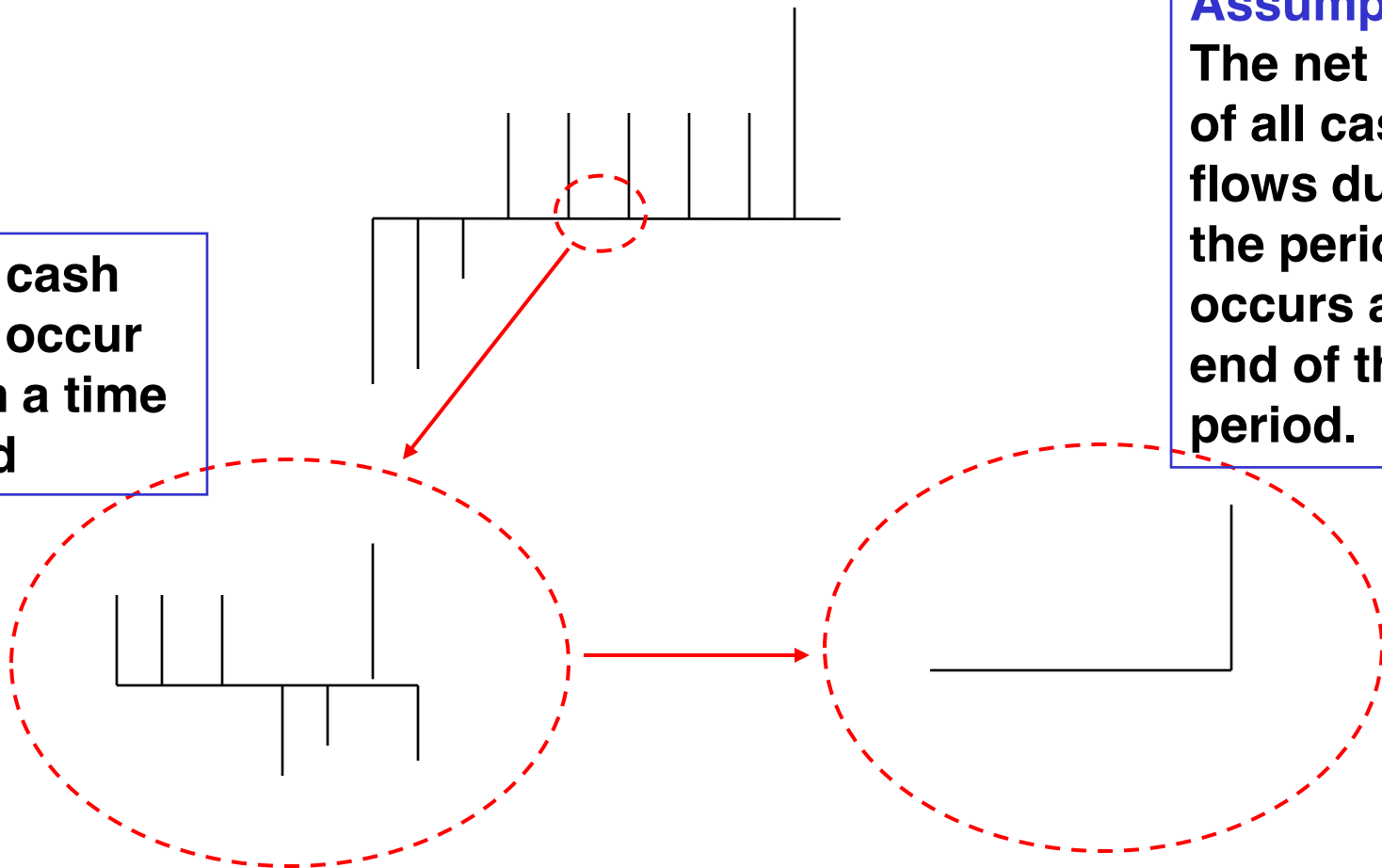
Cash flows in units of money (\$)



# Time value of money

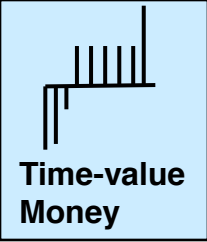
## Cash flow diagram and analysis

Many cash flows occur within a time period



**Assumption:**  
The net sum of all cash flows during the period occurs at the end of the period.



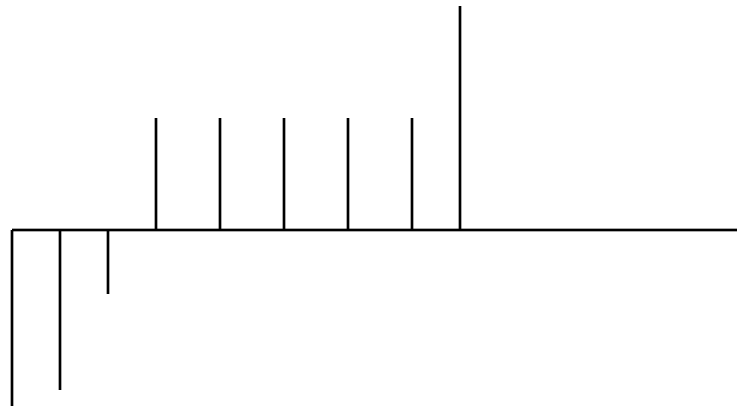


# Time value of money

We plot the end-of-period, or the cumulative cash flows

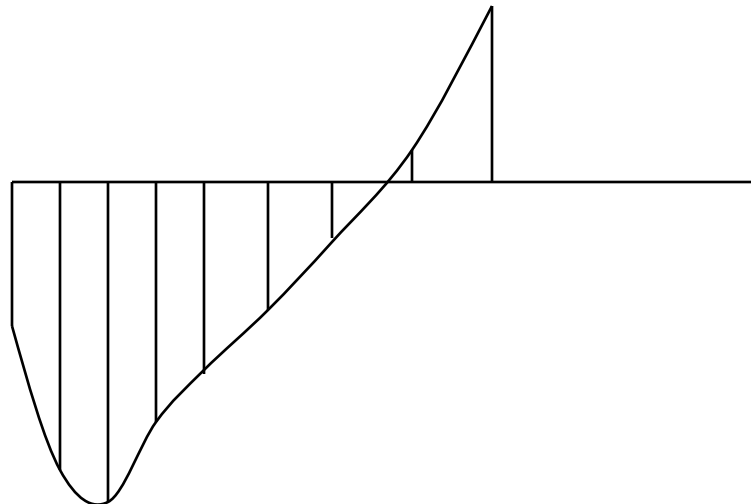
## Cash flow diagram

(at each period)

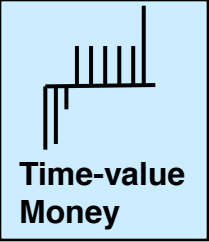


## Cumulative cash flow diagram

(just the cumulative sum of the above plot)



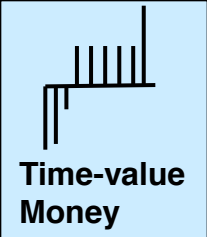
**We'll use both, with the top plot used more often.**



# Time value of money

Draw a cash flow diagram for life from age 10 to age 40 with periods of 1 year

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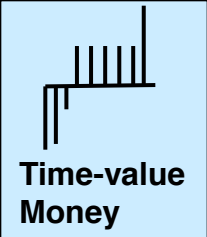


# Time value of money

**Key question:** Why is there a “time value of money”?

**Class exercise:** A family member asks you to lend her \$100. She promises to pay you exactly three years later. She will give you \$100 then.

***Is this a good financial proposition? Why?***



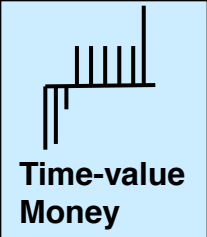
# Time value of money

## Why is there a “time value”?

- **The owner of money must defer its use**
- **The owner incurs risk**

**Thus, money in the future is worth less than money now.**

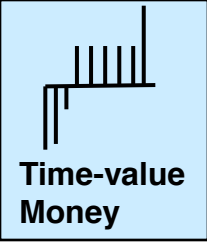
We must take this into account, as our employer’s money will almost always be spent over a long period of time.



# Time value of money

## How do we characterize time value?

- We use an **interest rate**, so that the effect of time is proportional to the total amount of money involved.

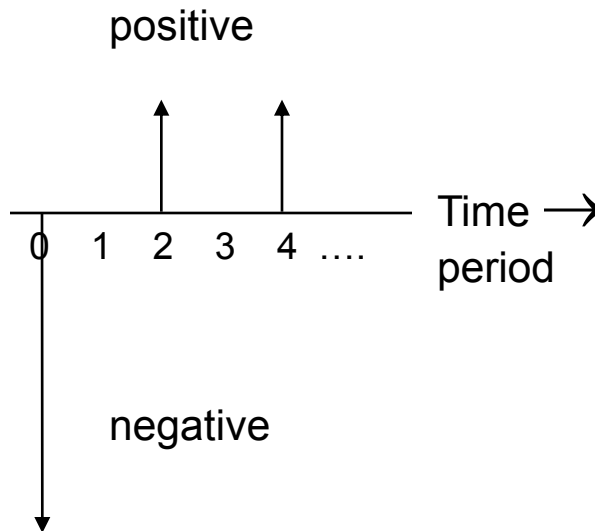


# Time value of money

We will use cash flow diagrams to summarize the behaviour of the system.

We need to calculate the value of all cash flows at the same time to make economic analyses.

Cash  
flow  
at each  
period  
(\$)

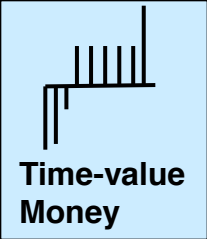


$P$  = present value (period = 0)

$F$  = future value (period > 0)

$i$  = interest rate

$n$  = number of periods  
between present and future



# Time value of money

## Example 1:

We would like a future amount  $F = \$1000$

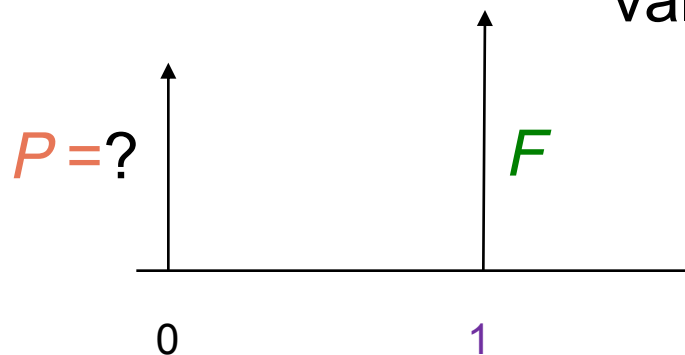
But we have only  $P = \$800$  to invest now.

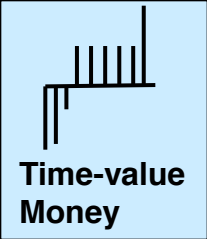
What interest rate is required to obtain  $F$  at  $n = 1$  year from now?

## Example 2:

We would like a future amount  $F = \$1000$  at  $n = 1$  year from now.

Given an interest rate  $i = 0.04$  [4%], how much should we invest today, called the present value,  $P$ ?





# Time value of money

Determine the relationships between  $P$  and  $F$  for  $n$  time periods, with compound interest rate  $i$



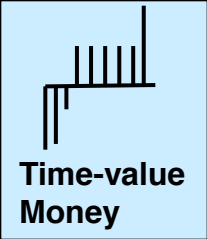
$$F_n = P (1 + i)^n$$

What is the present value of a revenue of  $F = \$1000$  at time  $n$  for each year  $n = 1, 2 \dots 10$  at 10% per year time value of money?

*Asked another way ...*

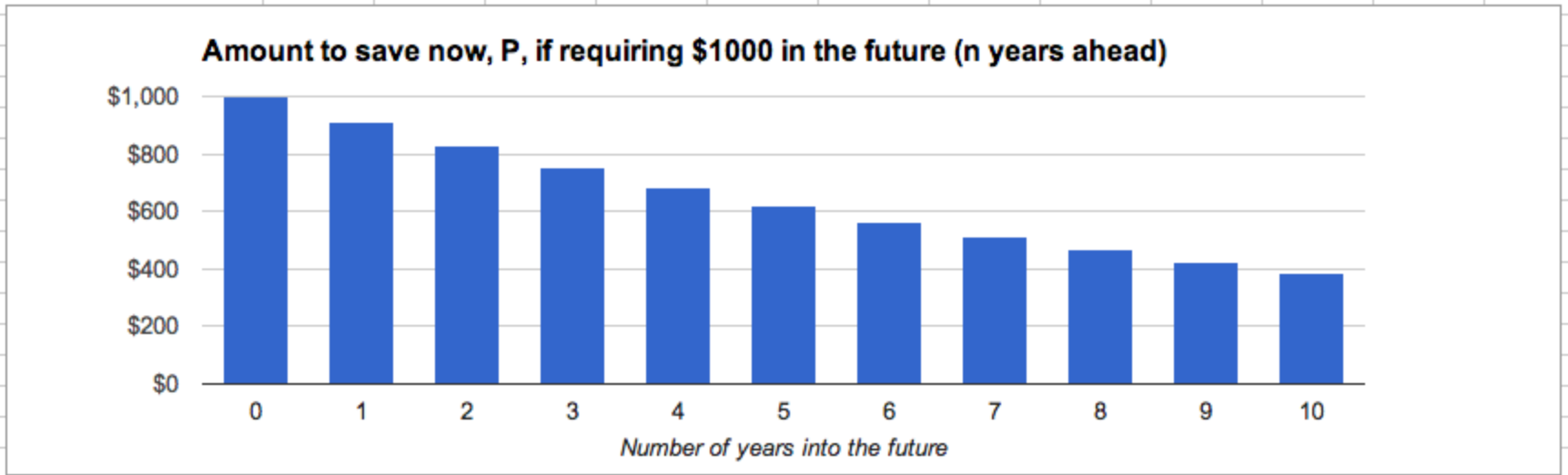
If you want to have  $F = \$1000$  in  $n = 1, 2, \dots 10$  years from now, how much do you have to invest right now, if interest rates remain at 10% per year?



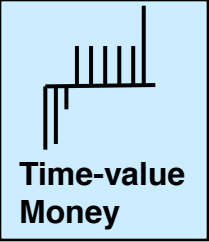


# Time value of money

Amount in the future (F)	\$1,000 (desired)										
Interest rate (i)	0.1										
Period, n	0	1	2	3	4	5	6	7	8	9	10
Present value required (P)	\$1,000	\$909	\$826	\$751	\$683	\$621	\$564	\$513	\$467	\$424	\$386

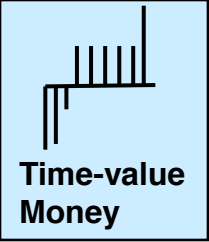


*Interpretation* : \$621 right now ( $n=0$ ) has the equivalent worth of what \$1000 will have 5 years ( $n=5$ ) from now, at interest rates of 10%.



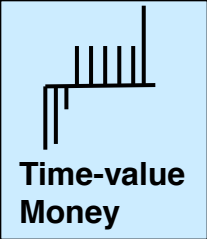
# Time value of money

**IGNORE**



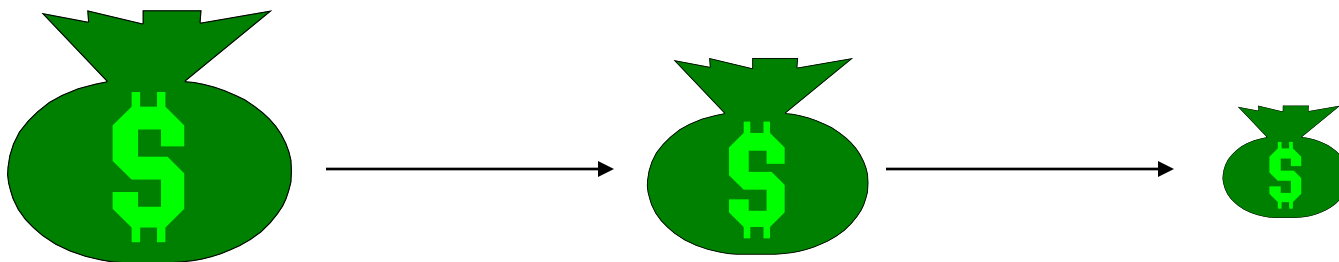
# Time value of money

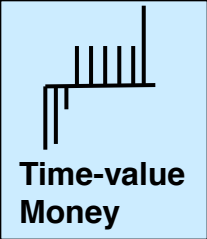
**IGNORE**



# Time value of money

- Since money has a time value, money in the future has less value. We will characterize this decrease with the “time value of money”.
- For a worthwhile investment, the net income in the future must be greater than the original expense.



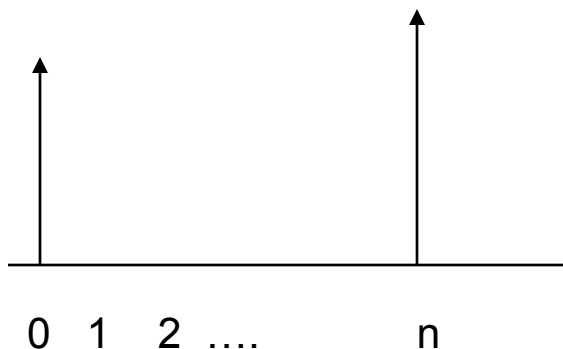


# Time value of money

**Associated use of interest rates:** When we place money in the bank, the bank increases the amount in our account according to an interest rate. This is payment for the bank using our money.

**Initial  
balance**

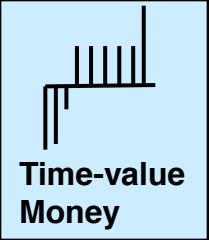
**Future  
balance = ?**



How do we calculate the future amount in our account?

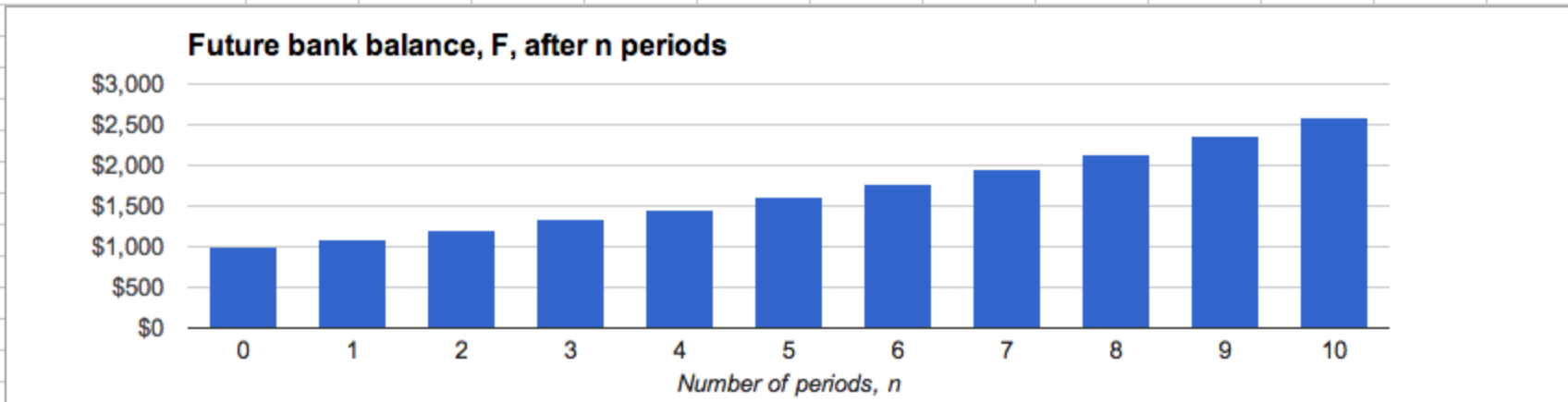
$$\text{Future balance} = P (1 + i)^n$$

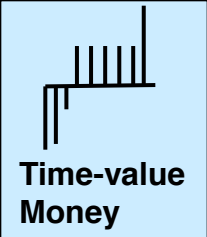
What is the amount in your account ten years after depositing \$1000 at 10% per year interest rate?



# Time value of money

<b>Amount invested (P)</b>	\$1,000										
<b>Interest rate (i)</b>	0.1										
<b>Period, n</b>	0	1	2	3	4	5	6	7	8	9	10
<b>Bank balance, F</b>	\$1,000	\$1,100	\$1,210	\$1,331	\$1,464	\$1,611	\$1,772	\$1,949	\$2,144	\$2,358	\$2,594



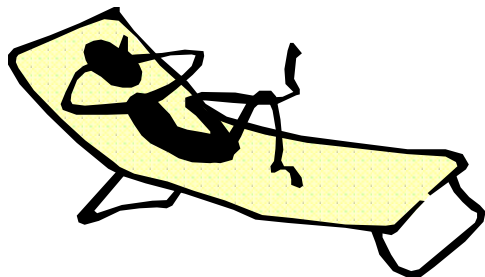


# Time value of money

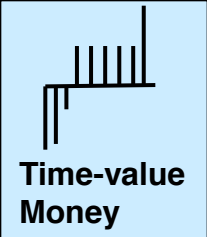
If you want to get rich, just invest and **wait**

Invest \$10,000/yr at 5% is worth after 35 years: \$ 948,000  
after 40 years: \$ 1,268,000  
after 45 years: \$ 1,677,000\*

\* This is close to the number we discussed at tutorial on Monday

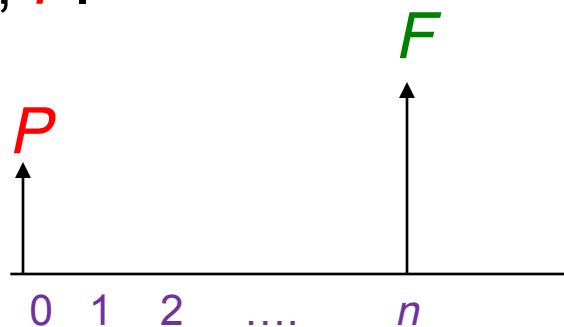


*“Compound interest is the eighth wonder of the world. He who understands it, earns it ... he who doesn't ... **pays it.**” – Albert Einstein*



# Time value of money

We can consider inflation,  $i$ , in a similar way. An amount of money in the future ( $F$ ), is worth less than in the present,  $P$ .



$$F_n = P (1 + i)^n$$

*Asked another way ...*

What is the **present value** of  $F = \$1000$  at time =  $n$

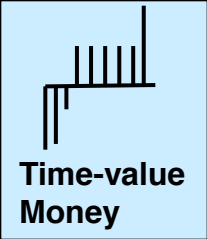
for each year ( $n = 1$  to  $10$ )

at **10% per year time** value of money?

In  $n = 1, 2, \dots, 10$  years from now you discover  $F = \$1000$  under your mattress, and you can go buy goods with those dollars.

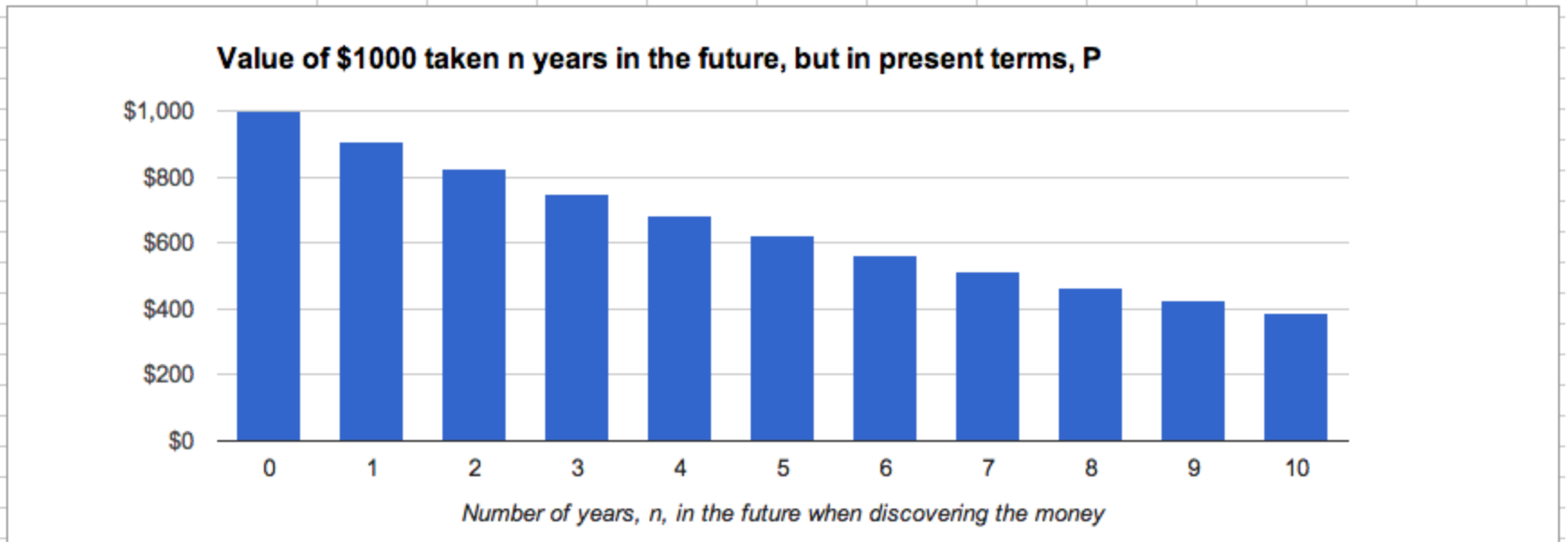
How much would those same goods have cost, **in today's dollars** if **inflation was 10% per year**?



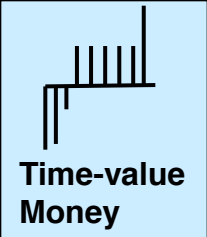


# Time value of money

Amount discovered later (F)	\$1,000											
Inflation rate (i)	0.1											
Period, n	0	1	2	3	4	5	6	7	8	9	10	
Value in present terms (P)	\$1,000	\$909	\$826	\$751	\$683	\$621	\$564	\$513	\$467	\$424	\$386	



*Interpretation* : If TVM (inflation) = 10%, then consider that something worth \$424 now is what you'll have to pay \$1000 for in 9 years from now.



# Time value of money

Economic and financial indicators

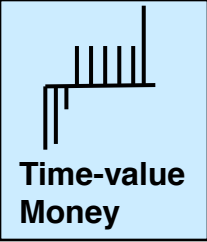
The Economist September 7th 2013

## Interest rates

### Economic data (2)

% change on year ago

	Current-account balance		Budget balance % of GDP 2013†	Interest rates, % 10-year gov't bonds, latest	Currency units, per \$	
	latest 12 months, \$bn	% of GDP 2013†			Sep 4th	year ago
United States	-425.7 Q1	-2.7	-4.0	2.89	-	-
China	+211.6 Q2	+2.1	-2.1	3.94 <sup>\$\$</sup>	6.12	6.35
Japan	+54.1 Jun	+1.2	-8.3	0.78	99.5	78.4
Britain	-96.7 Q1	-2.7	-7.6	2.95	0.64	0.63
Canada	-59.6 Q2	-3.1	-2.8	2.72	1.05	0.99
Euro area	+247.1 Jun	+1.8	-3.3	1.94	0.76	0.80
Austria	+9.7 Q1	+2.4	-3.0	2.37	0.76	0.80
Belgium	-8.7 Mar	-0.7	-3.1	2.82	0.76	0.80
France	-46.4 Jun	-2.1	-4.2	2.53	0.76	0.80
Germany	+244.5 Jun	+6.6	+0.3	1.94	0.76	0.80
Greece	-3.4 Jun	-0.8	-4.7	10.46	0.76	0.80
Italy	+5.9 Jun	+0.6	-3.5	4.42	0.76	0.80
Netherlands	+85.1 Q1	+8.1	-3.8	2.35	0.76	0.80
Spain	+8.7 Jun	+0.7	-7.2	4.43	0.76	0.80
Czech Republic	-3.8 Q2	-1.9	-2.9	2.58	19.5	19.8
Denmark	+18.2 Jun	+5.2	-2.6	2.13	5.65	5.93
Hungary	+3.0 Q1	+1.7	-3.0	6.59	228	227
Norway	+64.6 Q2	+12.9	+13.0	3.15	6.07	5.79
Poland	-9.3 Jun	-2.4	-3.9	4.80	3.24	3.34
Russia	+47.9 Q2	+2.5	-0.6	7.94	33.3	32.3
Sweden	+32.0 Q2	+7.1	-1.4	2.57	6.60	6.71
Switzerland	+77.6 Q1	+11.6	+0.2	1.18	0.94	0.96
Turkey	-53.6 Jun	-6.8	-2.2	10.02	2.06	1.82
Australia	-49.4 Q2	-3.1	-1.3	4.02	1.09	0.98
Hong Kong	+5.2 Q1	+0.9	+2.0	2.47	7.76	7.76
India	-87.8 Q1	-4.5	-5.1	8.39 <sup>†††</sup>	67.0	55.6
Indonesia	-28.8 Q2	-2.4	-2.9	na	11,065	9,571
Malaysia	+14.2 Q2	+5.8	-4.3	3.98	3.28	3.11
Pakistan	-2.3 Q2	-1.0	-8.8	12.10 <sup>†††</sup>	105	94.8
Singapore	+49.9 Q2	+18.3	+0.7	2.65	1.27	1.25
South Korea	+59.8 Jul	+3.5	+0.5	3.64	1,095	1,133
Taiwan	+52.8 Q2	+11.7	-1.9	1.73	29.8	29.9

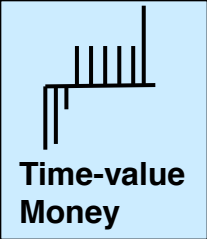


# Time value of money

**Class exercise:** Your bank account is the “system”. You have an initial revenue of \$4,000 and the following monthly revenues and expenditures, and the bank pays 5% interest per month.

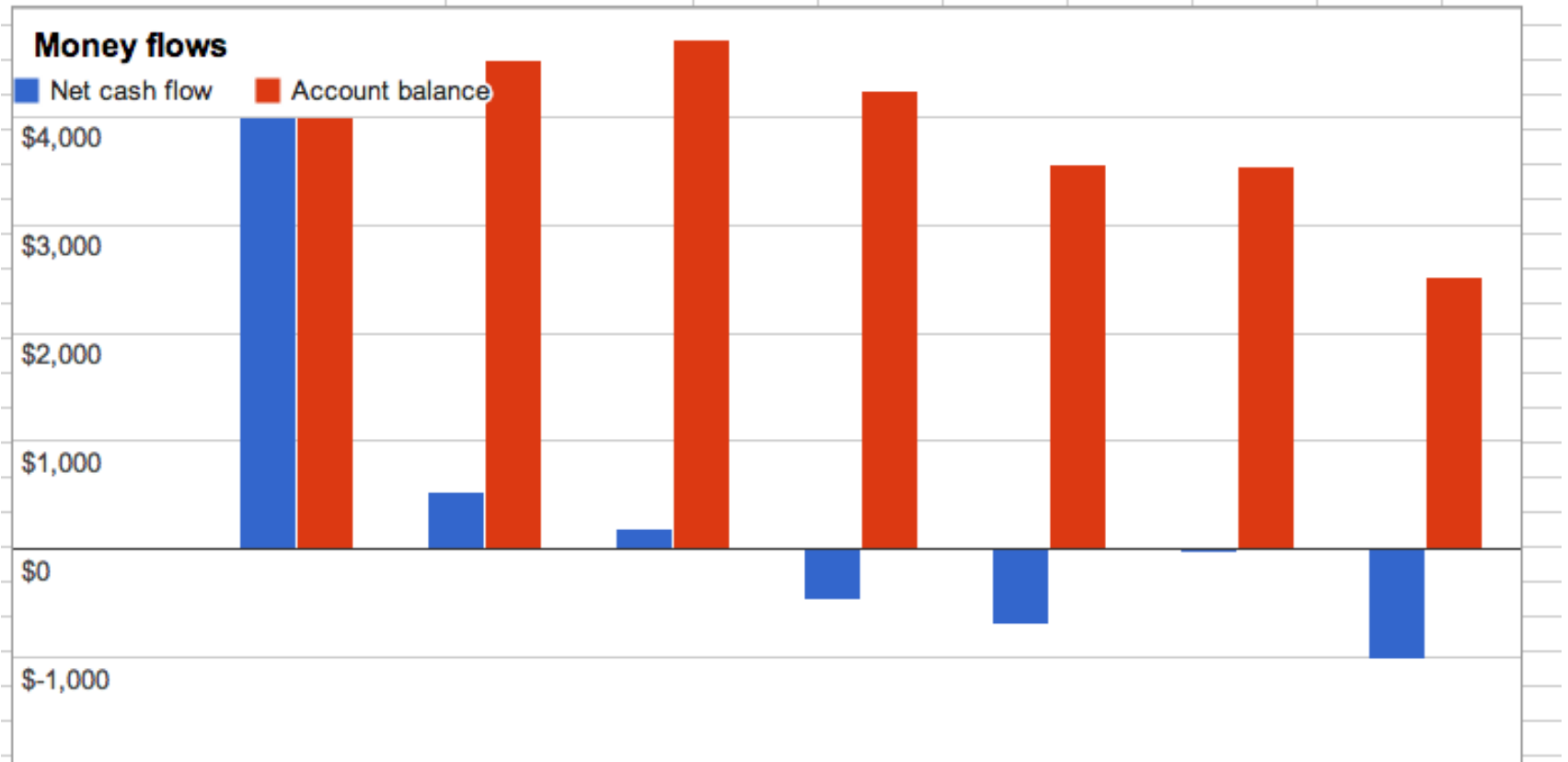
Plot the monthly balance *and* cash flow diagram for your bank account.

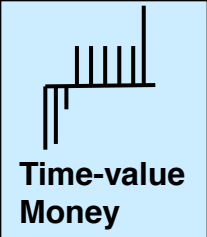
	Month	0	1	2	3	4	5	6
Revenues		\$4,000	\$530	\$530	\$0	\$0	\$0	\$0
Expenses		\$0	-\$200	-\$570	-\$700	-\$900	-\$200	-\$1,200



# Time value of money

	Month	0	1	2	3	4	5	6
Revenues	A	\$4,000	\$530	\$530	\$0	\$0	\$0	\$0
Expenses	B	\$0	-\$200	-\$570	-\$700	-\$900	-\$200	-\$1,200
Interest earned at 5% per month	$C = 0.05 \times E(n-1)$		\$200	\$227	\$236	\$213	\$178	\$177
Net cash flow	$D = A + B + C$	\$4,000	\$530	\$187	-\$464	-\$687	-\$22	-\$1,023
Account balance	$E = D + E(n-1)$	\$4,000	\$4,530	\$4,717	\$4,252	\$3,565	\$3,543	\$2,520



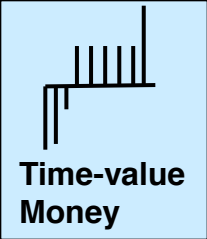


# Time value of money

**Now, let's relate the banking interest to the time value of money**

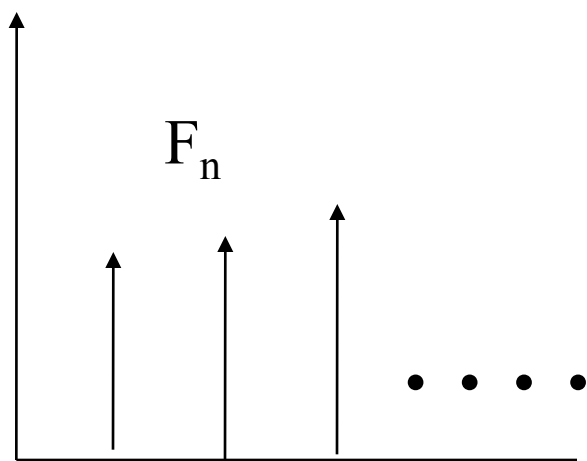
**Class exercise:** You deposit \$5000 in a bank account with an annual compound interest rate  $i^*$ . The time value of money is described by an interest rate  $i'$  (inflation rate).

Calculate the present value of the bank account after  $n$  years.



# Time value of money

$$C_0 = 5000$$



$$F_n = C_0 (1 + i^*)^n$$

Interest earned  
on the investment

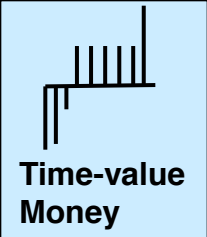
$$P = \frac{F_n}{(1 + i')^n}$$

Present value of  
the investment

$$P = C_0 \frac{(1 + i^*)^n}{(1 + i')^n}$$

What is the result if  $i^* = i'$ ?

How do we use this result to interpret the time-value of money?

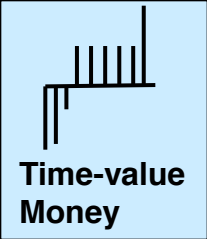


# Time value of money

## Class exercise

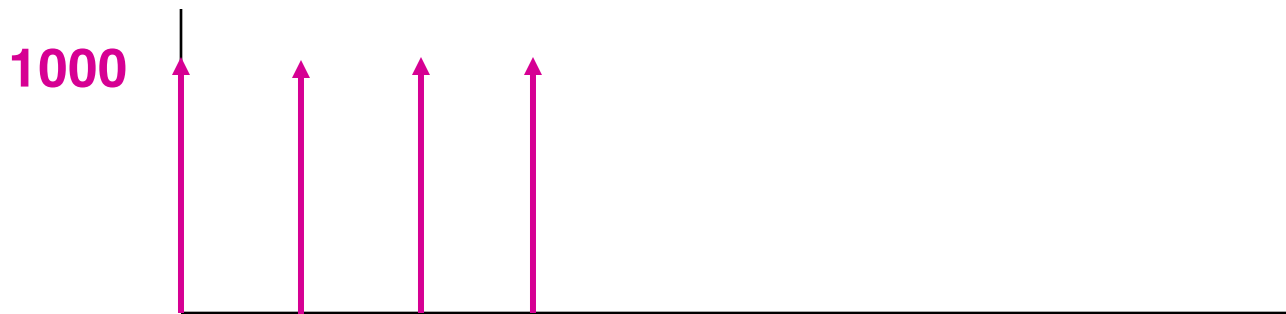
You have an income of \$1000 per year for each of the 4 years of your undergraduate studies.

- Draw a cash flow diagram
- Determine the value for this income in the beginning of the first year when the inflation rate (time value of money) is 10%.



# Time value of money

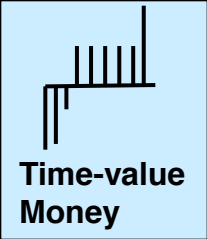
## Class exercise



		Inflation rate, $i = 0.1$			
Period	$n$	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
Cash flow in the period	$F_n$	\$1,000	\$1,000	\$1,000	\$1,000
Cash flow in present value terms	$P$	\$1,000	\$909	\$826	\$751
Cumulate cash flow in present value terms		<b>\$1,000</b>	<b>\$1,909</b>	<b>\$2,736</b>	<b>\$3,487</b>

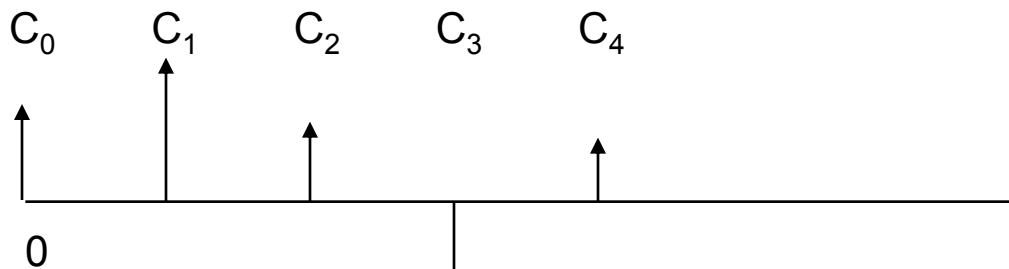
*Interpretation:* You could have replaced the cash flow with one revenue of \$3487 at time period 0, that earned interest at 10%. Then make \$1000 withdrawals in each year from the bank account. The balance will be \$0 after the last withdrawal. Prove this interpretation for yourself in a spreadsheet.





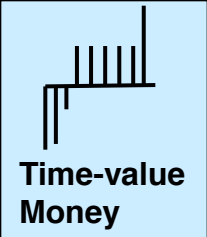
# Time value of money

**Look ahead:** We will be expressing values for different investments at the **same time** period for the purpose of comparison.



$$P = \frac{C_0}{(1+i)^0} + \frac{C_1}{(1+i)^1} + \frac{C_2}{(1+i)^2} + \frac{C_3}{(1+i)^3} + \frac{C_4}{(1+i)^4} + \dots$$

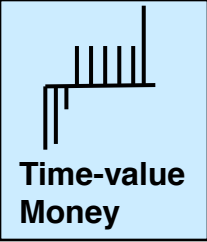
with  $C_n$  = cash flow at period  $n$  with a TVM rate of  $i$



# Time value of money

## Some thoughts

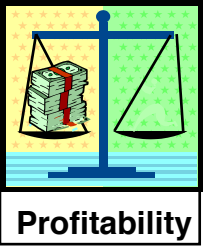
- **Interest factor tables:** Many tables are provided for relationships among  $P$ ,  $F$  and annuity values for specified interest rates and periods
- **Calculations:** Many projects have unequal cash flows. The time-value calculations are easily performed using spreadsheets like Excel.
- **Life-long Applications:** These concepts are useful for personal finances (mortgage rate, credit card borrowing, and so forth).



# Time value of money

## Group learning / Self-directed learning

1. Determine the meanings of **simple**, **compound**, **nominal**, **effective** and **continuous** interest.
2. How would the equations used in this section be changed if the interest rate depended on the period?
3. You have a balance of \$4,000 on your credit card which has an interest rate of 24% (nominal, **compounded monthly**). How much do you have to pay per month to maintain your balance at \$4,000? How much do you have to pay per month to clear your debt in one year?
4. What is the meaning of the term “usury”? What is the history of charging interest for loans? Read up on Sharia compliant finance (finance without charging interest on loans).
5. Investigate the **=PV ( )** and **=FV ( )** functions in spreadsheet software

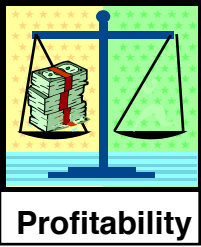


# Measures of profitability



1. Time value of money
2. Quantitative measures of profitability
3. Systematic comparison of alternatives
4. Estimation of costs

- We need a systematic method for **comparing** expenses and incomes at different times using the time value of money
- We need to compare the project profitability with a **minimum acceptable performance**
- Many measures are in use; we'll look at four.
  - Two are useful and commonly used by engineers
  - Two are **not recommended**, but are used in practice. We should know these as well.



# Measures of profitability

The following organizations and decisions are not “profit based”; do they need measures of profitability?



- Universities
- Charities
- Governments
- For-profit companies when involved in
  - safety projects
  - environmental projects



# Measures of profitability

## Examples for each category



- Universities – e.g. rent or purchase computers
- Charities - Invest in fund raising
- Governments - In-house or outsource tasks
- For-profit companies when involved in

- safety projects
- environmental projects

Find project that satisfies goals at the lowest cost



# Measures of profitability

## Example

We can invest money yielding a 15% annually compounded return.

Compared to that, would the following project be financially attractive?

i.e. should we invest, or just park our money and earn the 15%?

Period	Cash Flow (\$)
0	-91,093
1	20,000
2	40,000
3	40,000
4	40,000
5	30,000

Don't know how to estimate the costs?  
Don't worry, we will cover the topic soon.





# Measures of profitability

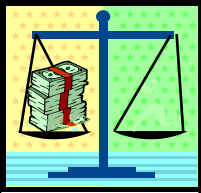
## Payback time

- This measure is often used as a “quick and dirty” measure of profitability
- We use it in our daily lives: how long does it take to pay back for ...(car, vacation, new cell phone, *etc*)
- Also called *Payout Time*
- Defined in units of time (e.g. months or years)

**The time for the cumulative cash flow to achieve a value of \$0**

Usually (and in this course), payback time does not consider interest.





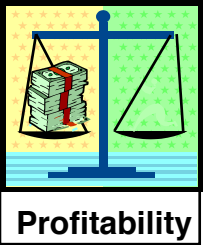
Profitability

# Measures of profitability

## Class exercise: Payback time

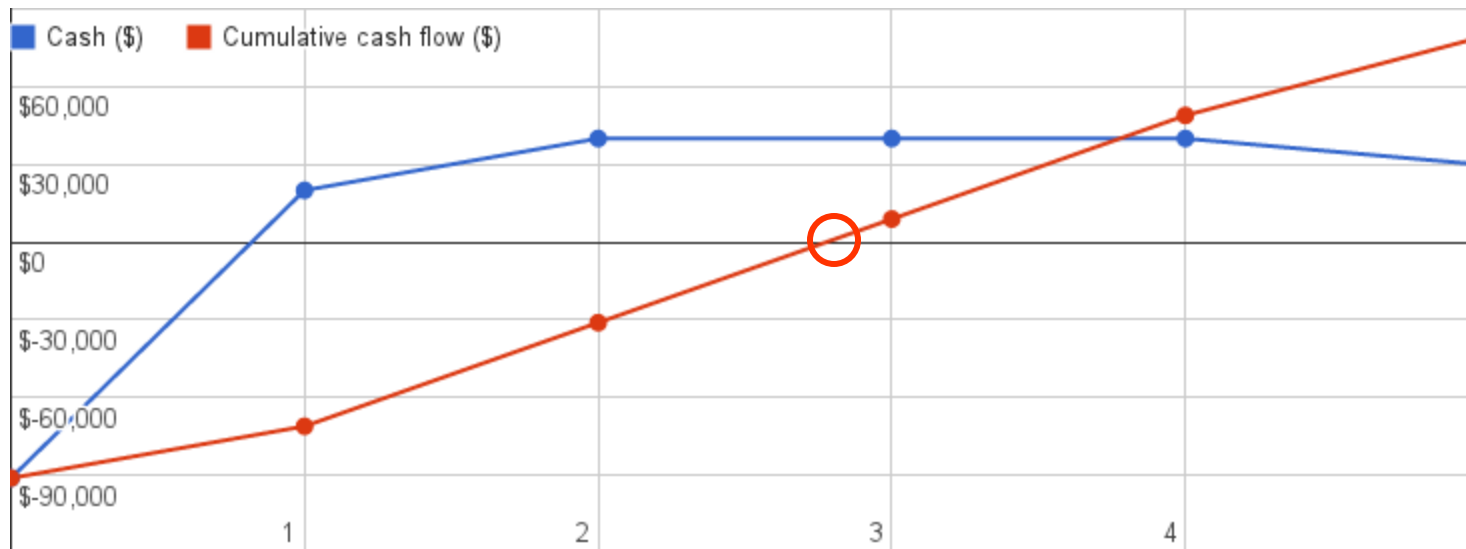
Determine the payback time for the cash flow defined in previous table

Period	Cash Flow (\$)
0	-91,093
1	20,000
2	40,000
3	40,000
4	40,000
5	30,000



# Measures of profitability

A plot (visual interpolation) used to determine the payback time





# Measures of profitability

- What is the **Payback time** for a project that involves an original investment of \$91,000 and provides an annual profit (positive cash flow) of \$34,000 per year over the first three years and no depreciation.

**Payback time** =  $91/34 \approx 2.7$  years [rough calc.]

*Same payback time as previous example, but different cash flows*

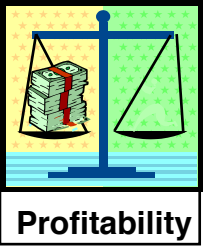
## Notes



- **No time value of money taken into account**
- Doesn't consider what happens after payback

**Not recommended!**

*Can be an effective screening tool though*



# Measures of profitability

## Return on original investment (ROI)

- Simple calculation
- $$\text{ROI} = \frac{\text{average annual profit}}{\text{fixed capital} + \text{working capital}}$$
- Expressed in units of percent per year

**What is fixed capital?  
What is working capital?**



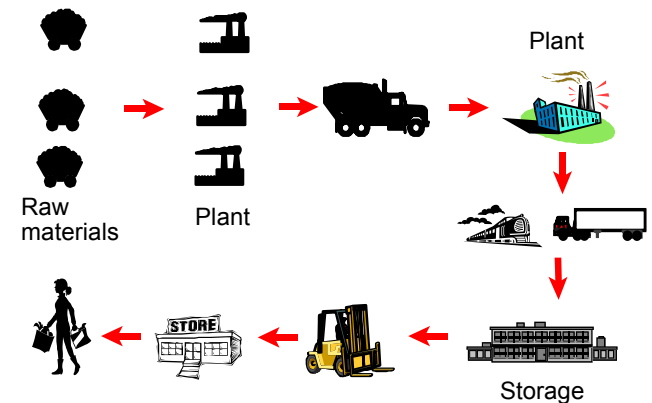


# Measures of profitability

## Working Capital

**Working capital is the difference between current assets and current liabilities. (Estimation given later in course.) Examples include:**

- Raw materials
- Work in progress (WIP), which is material part way through the production
- Supplies stored for manufacturing, e.g., catalyst
- Finished products in storage and transport that we still own
- Cash on hand to cover short-term expenses



**A key feature of working capital is that it can be recovered when the plant is shutdown.**



# Measures of profitability

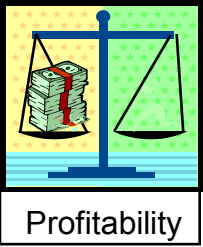
- Calculate the ROI for a project with fixed capital of \$91,000, no working capital, and an average annual profit of \$34,000.

$$\text{ROI} = 34/91 \times 100 \approx 34\%$$



**Does not consider time value of money**

**Not recommended!**



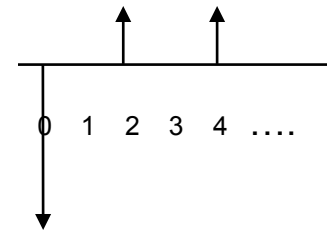
# Measures of profitability

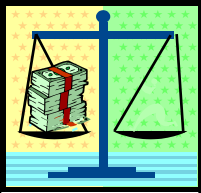
## Net Present Value (NPV) (NP worth)

- Explicitly expressed as a **specific value of money**
- Defined as present value of all cash flows
- Sum up these present values (i.e. “net” them up)
- For  $N$  compounding periods in the life of the project, with a net cash flow in each period of  $C_n$

recommended

$$\text{NPV} = \sum_{n=0}^N C_n (1+i)^{-n}$$





Profitability

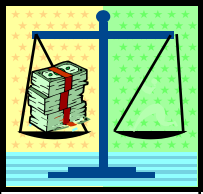
# Measures of profitability

## Class exercise: Net Present Value (NPV)

Period	Cash Flow (\$)	PV of cash flow (\$)
0	-91,093	
1	20,000	
2	40,000	
3	40,000	
4	40,000	
5	30,000	

Calculate the **NPV** for this project at 15% time value of money





Profitability

# Measures of profitability

## Class exercise: Net Present Value (NPV)

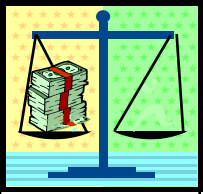
See the calculations below and *on the course website*

Payback time	Period	Cash (\$)	Present value (\$)	Cumulative sum of PV (\$)	Cumulative cash flow (\$)
	0	-\$91,093	-\$91,093	-\$91,093	-\$91,093
	1	\$20,000	\$17,391	-\$73,702	-\$71,093
Interest rate	2	\$40,000	\$30,246	-\$43,456	-\$31,093
0.15	3	\$40,000	\$26,301	-\$17,155	\$8,907
	4	\$40,000	\$22,870	\$5,715	\$48,907
	5	\$30,000	\$14,915	\$20,630	\$78,907

What does this value mean?

From prior exercise

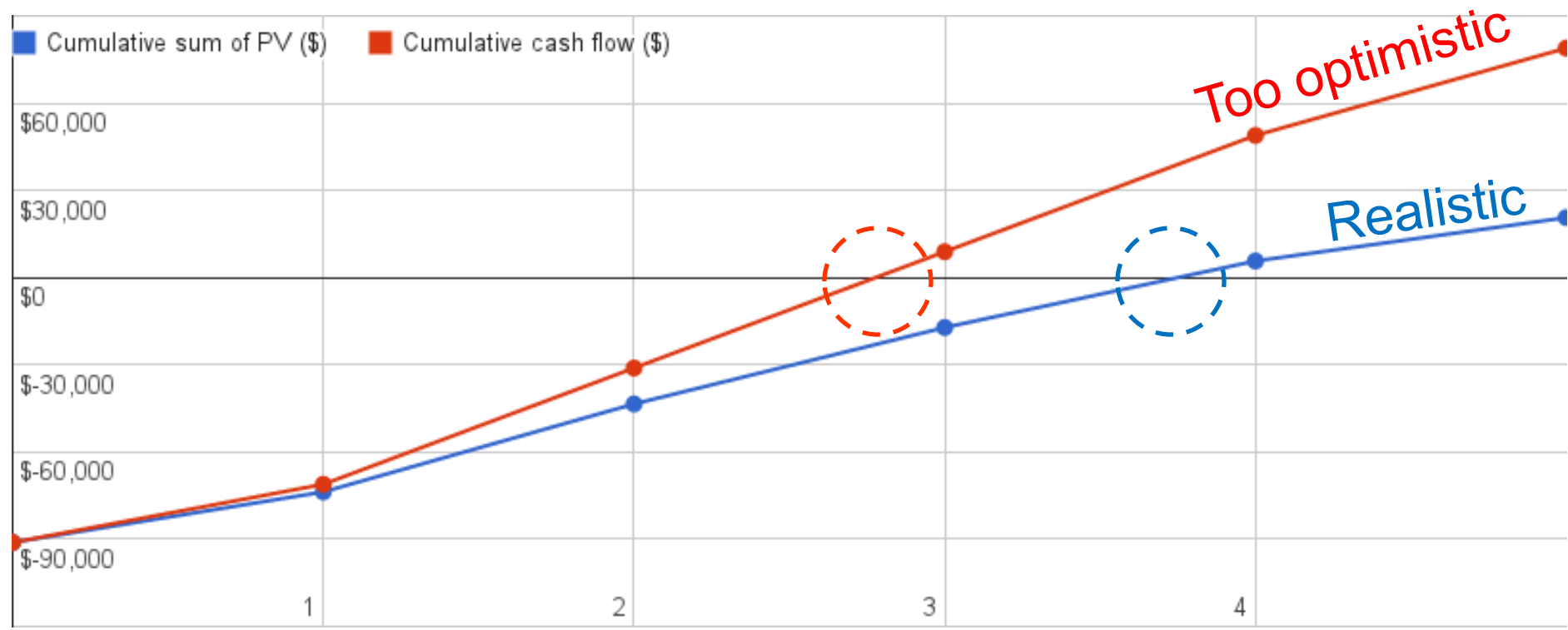
This approach considers time value of money explicitly. Important for projects of long duration, and in high deflationary environments.



Profitability

# Measures of profitability

## Class exercise: Net Present Value (NPV)



**Payback time** **not** taking time value of money into account is too optimistic.