

# Latent Variable Methods Course

## Learning from data

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# Image analysis aided by multivariate methods

We will look at

1. Multivariate image analysis (MIA)
2. Also show how classical image analysis tools are combined with multivariate tools, especially PLS

*Credits:* MACC group: particularly

- ▶ Honglu Yu
- ▶ J. Jay Liu

We will study some applications from their PhD theses today.

# References

- ▶ General image analysis book – Gonzalez and Woods: “Digital Image Processing”
- ▶ Good background: [Multivariate image analysis: A review with applications](#)
- ▶ General MIA technique: [Geladi and Grahn](#), 1997 book
- ▶ Paper: [Esbensen and Geladi](#) - Strategy of MIA

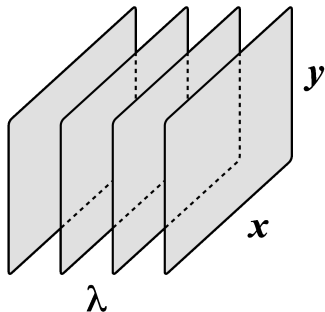
## *McMaster student applications:*

- ▶ Lumber, pulp, steel: [Manish Bharati's PhD thesis](#), 2002
- ▶ Snackfoods and flames: [Honglu Yu's PhD thesis](#), 2003
- ▶ Texture, wavelets, appearance monitoring and optimization: [June Liu's PhD thesis](#), 2004
- ▶ Medical images: [Mark-John Bruwer's PhD thesis](#), 2006
- ▶ Wheat grading with NIR: [Zheng Liu's Masters thesis](#), 2006
- ▶ Oats/groats detection: [Emily Nichols' Masters thesis](#), 2011

## Reasons for using image data

- ▶ Conventional sensors are expensive ( $\sim$  \$ 5000 or more), but digital cameras are relatively cheap
- ▶ Image sensors work well for solids-processing systems, which generally have fewer sensors available
- ▶ Cameras take non-destructive samples
- ▶ Can be implemented on-line
- ▶ Often picks up information not available from other sensors

## Conceptual storage of image data



The wavelength dimension (spectral dimension):

- ▶ 1 channel: grayscale
- ▶ 3 channels: colour image, RGB image
- ▶ multiple channels: hyperspectral image (e.g. NIR camera)

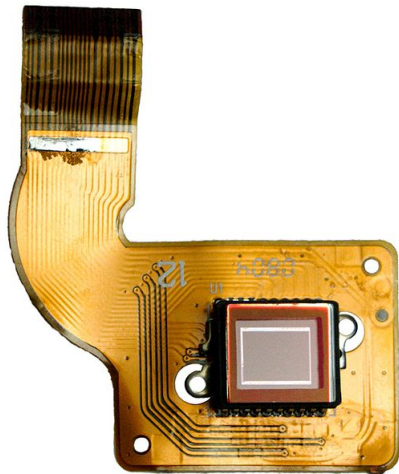
Tradeoff: *usually* we see lower spatial resolution as we acquire more channels, mainly due to cost of acquiring the additional wavelengths

# What do we want from our images

We will see examples where we:

- ▶ Monitor for process problems
- ▶ Make predictions from the image data
- ▶ Optimize and improve our processes

# How image data is acquired



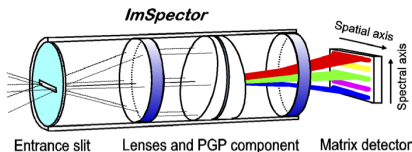
Charge-coupled device (CCD)  
senses the light.

Photons received  $\rightarrow$  voltage  $\rightarrow$   
digitized and stored.

Image source: [Wikipedia](#) (CC-BY-SA)

# How image data is acquired

NIR cameras (multiple channels) <sup>1</sup>

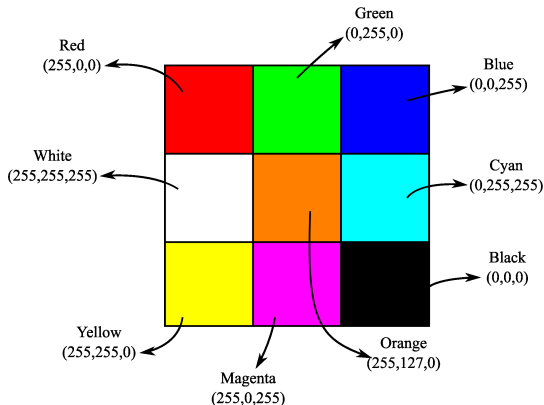


<sup>1</sup>From **Emily Nichols' thesis** (used with permission)

# Actual electronic storage of image data

Pixels in the image are stored as a sequence of (usually) integer values

- ▶ RGB pixel with (0, 0, 0) is a black pixel
- ▶ RGB pixel with (0, 0, 255) is a blue pixel
- ▶ RGB pixel with (255, 255, 255) is a white pixel



## Actual electronic storage of image data

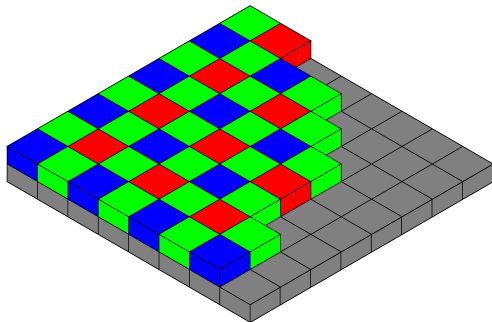
- ▶ Integers from 0 to 255 are stored in 8 bits (one byte) per pixel
- ▶ Medical images require greater spectral resolution: often use 2 bytes: 0 to 65535 (16 bits, but use only 10 or 12 bits)

### Tip

Never store your image data as JPEG ← uses lossy compression;  
rather store as PNG (lossless compression)

# Not all colour images are true colour

True colour images split the light, then use 3 CCD arrays for the **R**, **G**, and **B** channels.



Everyday colour images (cellphones, digital cameras) are in fact from a single CCD with a special array of colour filters: Bayer filter

The R, G, B layers we see in the digital file are algorithmically created from the single CCD matrix.

Image source: [Wikipedia](#) (CC-BY-SA)

## Viewing image data: grayscale

Computer monitors display RGB colour images.

Grayscale images displayed with equal weight in the 3 channels

Convert RGB to grayscale:

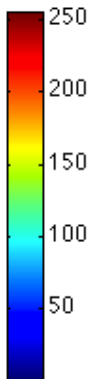
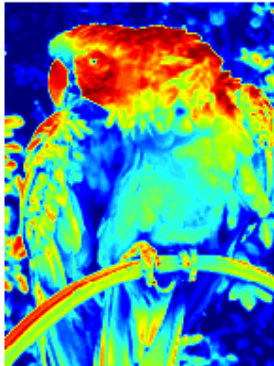
$$\text{Gray} = 0.3R + 0.59G + 0.11B$$

which approximates the human eye's weighting of our light-sensitive cones.



## Viewing image data: grayscale

A grayscale image has values between 0 and 255 – map each integer to a different colour: `colormap()`

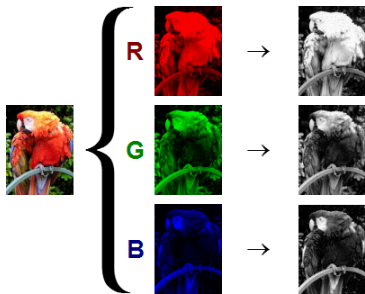


Creates a false-colour image from the grayscale image to help visualization. Many standard colour mappings are available.

## Viewing image data: colour

Show each RGB channel individually, in only that channel (middle), or as grayscale (right)

- ▶ Low integer values will be dark
- ▶ High integer values will be light

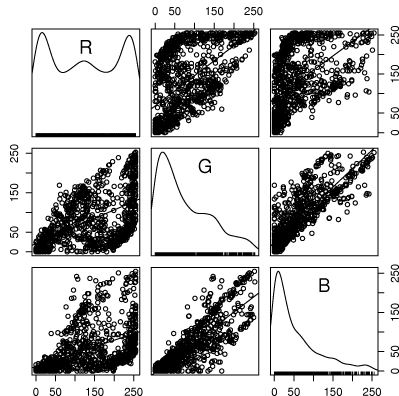


## Viewing image data

Images with 4 or more wavelengths: create a *false-colour* composite image by picking any 3 channels.

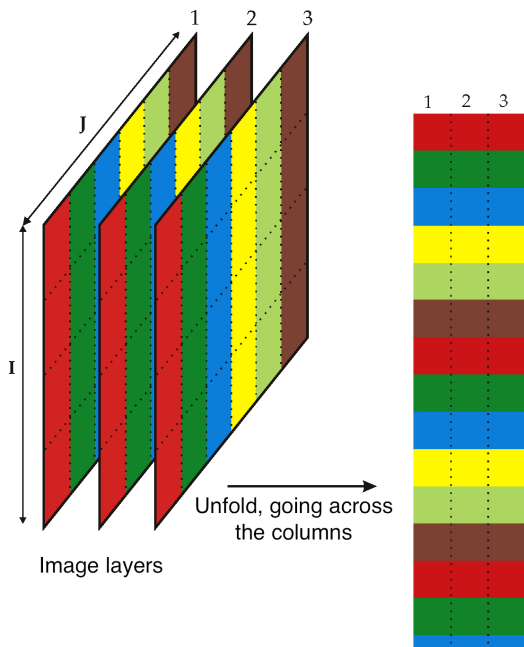
# Using PCA for multivariate image analysis (MIA)

Multiple channels in the images are often strongly correlated, especially for NIR images



Strong green and blue interaction

## Unfolding the image data

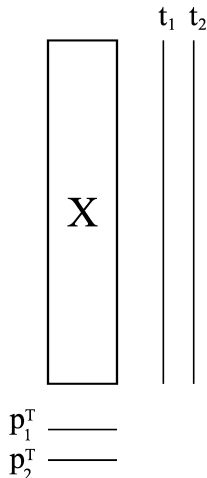


Other unfolding types possible, but this way makes sense to understand the correlation between the channels.

Sometimes called “*matricizing*” the image.

# Calculating the PCA model

Once  $\mathbf{X}$  is unfolded, we have a long, thin matrix.



Use the kernel PCA method on unfolded image data  $\mathbf{X}$ :

$$\mathbf{P} = \text{eigenvectors of } \mathbf{X}^T \mathbf{X}$$

$$\mathbf{T} = \mathbf{X} \mathbf{P}$$

We may choose to use all  $A = K$  eigenvectors in  $\mathbf{P}$ , or when  $K$  is large – e.g. NIR images – we only retain the first few ( $A < K$ ):

$$\mathbf{X} = \mathbf{T} \mathbf{P}^T + \mathbf{E}$$

$$\mathbf{E}_a = \mathbf{X}_{a-1} - \mathbf{t}_a \mathbf{p}_a^T$$

$$\mathbf{X}_{a-1} = \text{original image data}$$

We don't usually preprocess the image (there isn't much difference if you do, but it is expensive).

# Using the PCA model

We can investigate the model in the usual way:

- ▶ loadings
- ▶ binary score plots, e.g  $\mathbf{t}_1$  vs  $\mathbf{t}_2$ : not practical
- ▶ SPE and Hotelling's  $T^2$  (not practical)

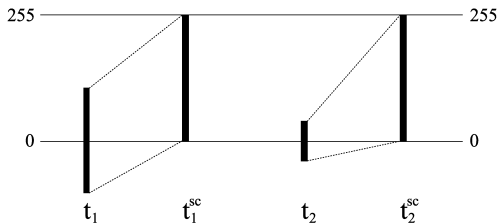
What do we want from the scores and SPE anyway?

## Refolding the scores and residuals

Notice that each score  $\mathbf{t}_a$  and each residuals in SPE have the same shape as the original image. It is natural to display these as grayscale images.

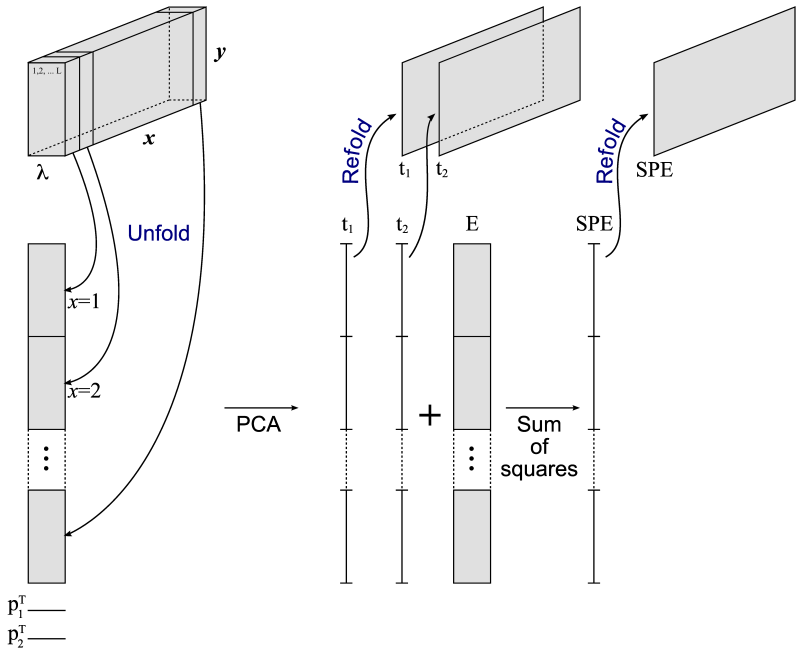
- grayscale displays require values between 0 and 255

The calculated  $\mathbf{t}_a$  values have mean of zero. To display  $\mathbf{t}_a^{\text{sc}}$  in the software:



Rather display  $\mathbf{t}_a^{\text{sc}}$  with colour coding.

$$\mathbf{t}_a^{\text{sc}} = \left\lfloor 255 \cdot \frac{\mathbf{t}_a - t_a^{\min}}{t_a^{\max} - t_a^{\min}} \right\rfloor$$



# Binary score plots

After scores are scaled as integers between 0 and 255:

- ▶ fewer unique combinations of  $t_h$  vs  $t_v$
- ▶ show it as a 2D histogram instead
  - ▶  $h$  = horizontal axis
  - ▶  $v$  = vertical axis
  - ▶ height: use a colour proportional to number of pixels at  $(t_h, t_v)$  combination

Let's take a look at an example ...

# An example

Original RGB image



Image is included with  
MACCMIA

You can follow along in class

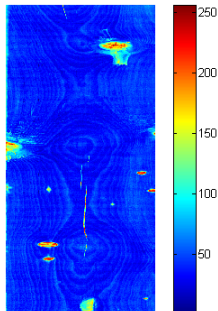
More details in [Bharati,  
MacGregor and Tropper](#)

# The first component

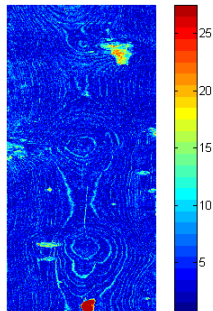
The RGB image from the data file



Intensity image of principal component 1



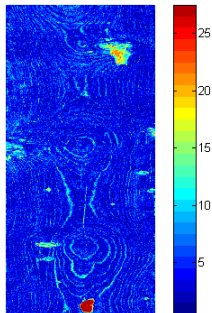
Residual intensity image after 1 PC extracted



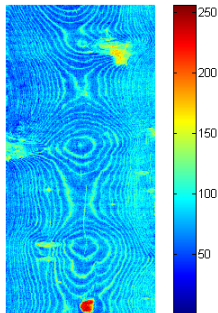
$$p_1 = [-0.72, -0.59, -0.37]$$

## The second component

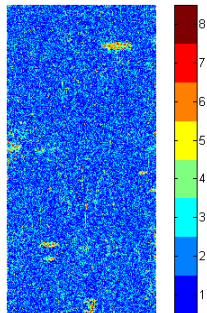
Residual intensity image after 1 PC extracted



Intensity image of principal component 2

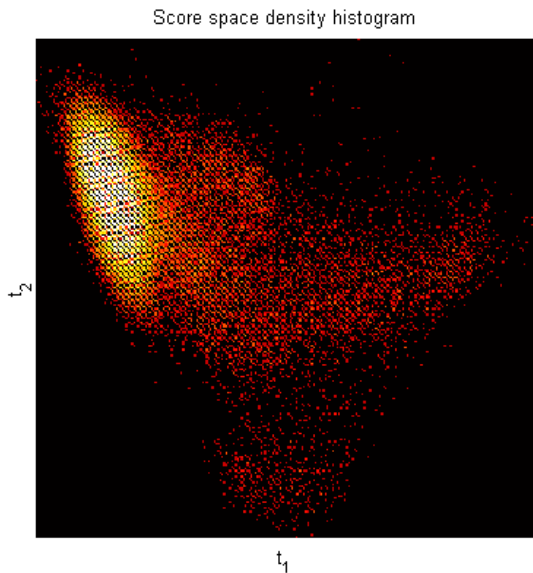


Residual intensity image after 2 PC's extracted



$$p_2 = [0.53, -0.13, -0.84]$$

# Score plot

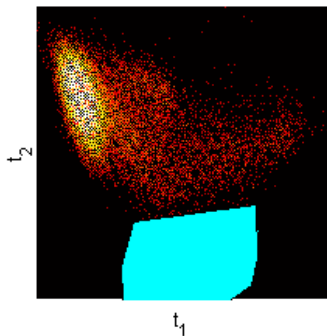


# Masking in the score space

Image, with mask mapped back

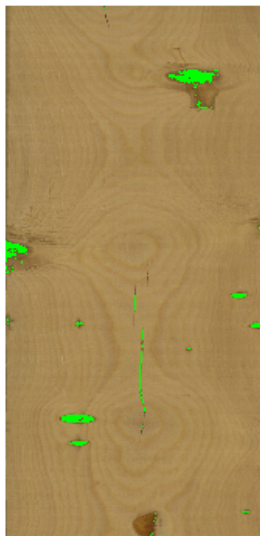


Score space density histogram

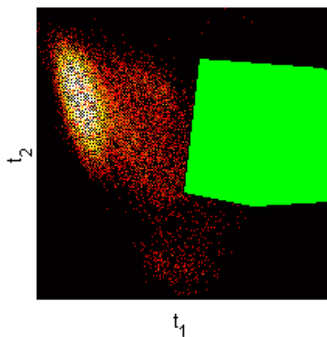


# Masking in the score space

Image, with mask mapped back



Score space density histogram



## Using the PCA model on a new image pixel

We have an existing PCA model; consists of

- ▶ loadings  $\mathbf{P}$ , an  $K \times A$  matrix
- ▶  $K$  = number of wavelengths (channels)
- ▶  $t_a^{\min}$  and  $t_a^{\max}$
- ▶ one or more score-space masks (store the vertices)

A new image (of different size, even as small as a single pixel):

- ▶ Unfold the image as  $\mathbf{X}_{\text{new}}$
- ▶  $\mathbf{X}_{\text{new}}$  has as many rows as there are pixels; has  $K$  columns
- ▶  $\mathbf{T}_{\text{new}} = \mathbf{X}_{\text{new}} \mathbf{P}$  = scores (has  $A$  columns)
- ▶ For each column in  $\mathbf{T}_{\text{new}}$ :
  - ▶  $\mathbf{t}_{a,\text{new}}^{\text{sc}} = \left\lfloor 255 \cdot \frac{t_{a,\text{new}} - t_a^{\min}}{t_a^{\max} - t_a^{\min}} \right\rfloor$
- ▶ For each row in  $\mathbf{T}_{a,\text{new}}^{\text{sc}}$  check whether the new score values lie under a mask

## Other hints and techniques

We have a problem if image space objects are spread across more than 2 different scores:

- Resolved using **automatic masking with support vector machines**

Building a model from multiple images?

1. Just paste images side-by-side to create a big, single image
2. More elegantly:

$$\left(\mathbf{x}^T \mathbf{x}\right)_{\text{composite}} = \left(\mathbf{x}^T \mathbf{x}\right)_1 + \left(\mathbf{x}^T \mathbf{x}\right)_2 + \dots + \left(\mathbf{x}^T \mathbf{x}\right)_{\text{last image}}$$

Calculate eigenvectors from  $\left(\mathbf{x}^T \mathbf{x}\right)_{\text{composite}}$

# Principle of Multivariate Image Analysis

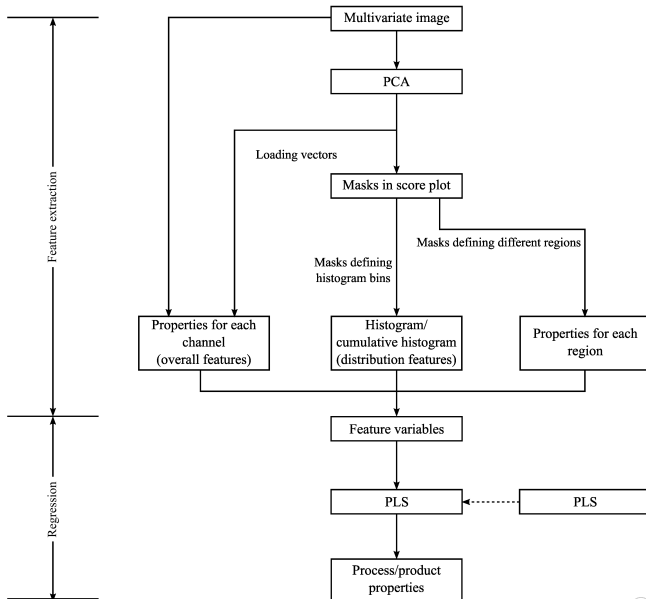
## Multivariate Image Analysis (MIA)

Objects in the image with similar *spectral* signature, no matter where they are *spatially*, will appear clustered in the scores.

- ▶ There is a many-to-one mapping between the image space and score space
  - ▶ One location in the score space can appear in many spatial locations in the image space.

As a result, MIA is most useful when spatial information is not too important.

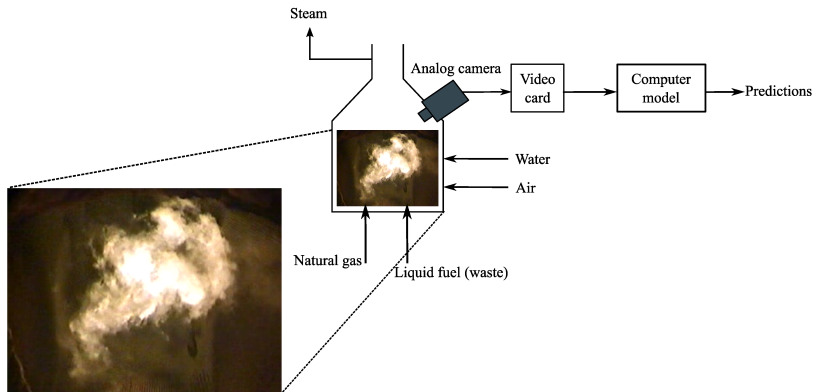
# General strategy for MIA



# Applications of image analysis

1. Flame monitoring and prediction (in depth)
  - ▶ Credit: thanks to Honglu Yu for permission to use her slides
  - ▶ PhD thesis
2. Snack food seasoning prediction, and monitoring (brief)
  - ▶ Credit: Honglu Yu
3. Automated judging steel sheets
  - ▶ Credit: thanks to J. Jay Liu for permission to use his slides
  - ▶ PhD thesis

# Flame: system overview

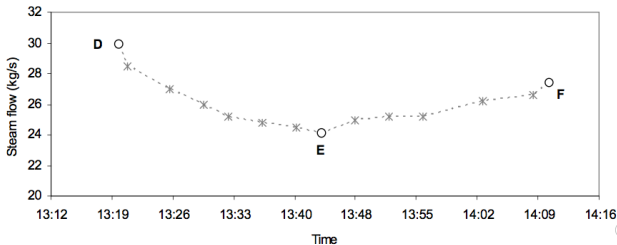
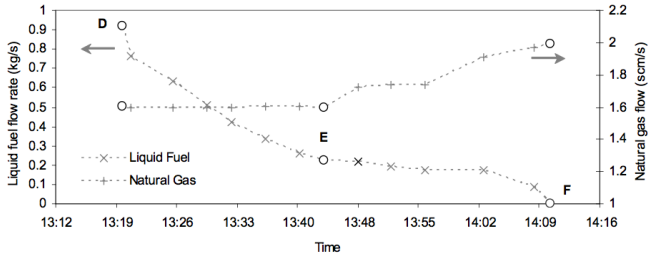


- ▶ Liquid waste stream has energy content
- ▶ Perhaps predict steam flow rate from image?

# Data available

Process data: average value over  $\sim 3$  minute interval

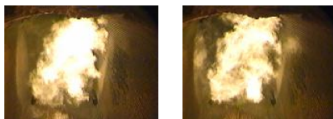
- ▶ steam flow (y variable)
- ▶ natural gas and waste stream flows



## Data available

Video data: images digitized 1 second apart. Examples at D, E, F:

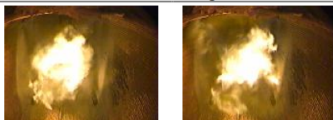
D



Liquid fuel: 0.917 kg/s  
Natural Gas: 1.61 scm/s  
Steam: 30.6 kg/s

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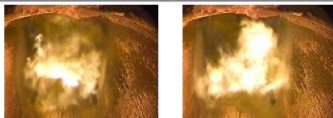
E



Liquid fuel: 0.225 kg/s  
Natural Gas: 1.58 scm/s  
Steam: 24.1 kg/s

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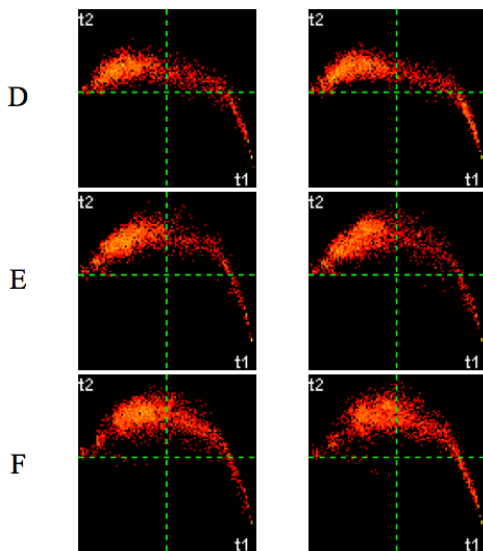
F



Liquid fuel: 0 kg/s  
Natural Gas: 2 scm/s  
Steam: 27.2 kg/s

► Each frame is  $120 \times 160 \times 3 = 56$  kilobytes

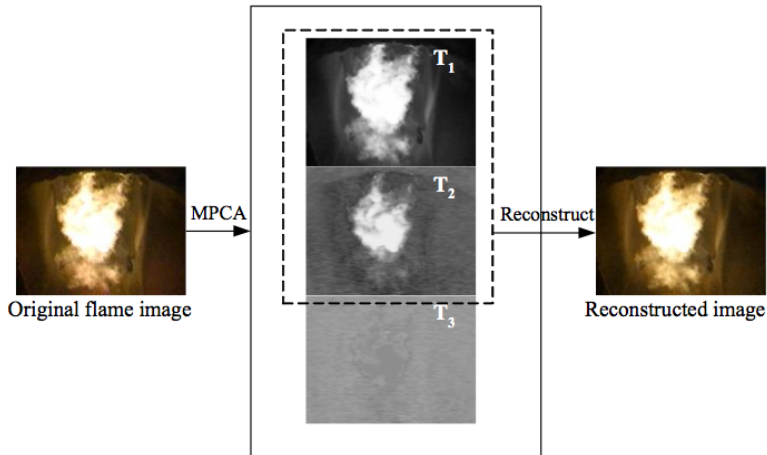
## PCA on the flame images



*Show video*

## PCA notes

- ▶ Use only 2 components: third PC mainly noise
- ▶ Reconstruct the denoised image:  $\hat{\mathbf{X}} = \mathbf{t}_1\mathbf{p}_1^T + \mathbf{t}_2\mathbf{p}_2^T$



## Important note

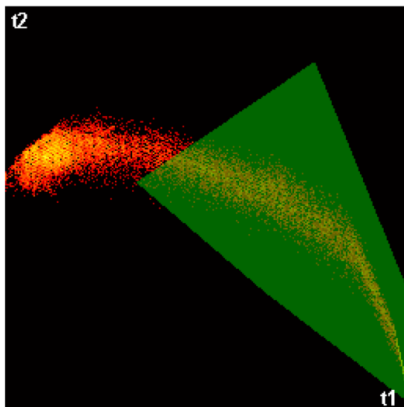
Based on the video and previous images:

- ▶ The image space *changes rapidly* from frame-to-frame
- ▶ Classical approaches to flame monitoring extract features directly from the image. For example:
  - ▶ perimeter
  - ▶ area
  - ▶ sphericity
  - ▶ luminosity
  - ▶ entropy (sum of absolute values)
  - ▶ maximum and minimum intensity
- ▶ The score space is much more stable
  - ▶ Use the score space to extract features

# First segment the images

**Def:** *segmentation* – to divide the image into 2 or more regions

- ▶ Separate “flame” from “non-flame”
- ▶ Hard and slow to do in the image space (try it!)
- ▶ Much easier in the score space



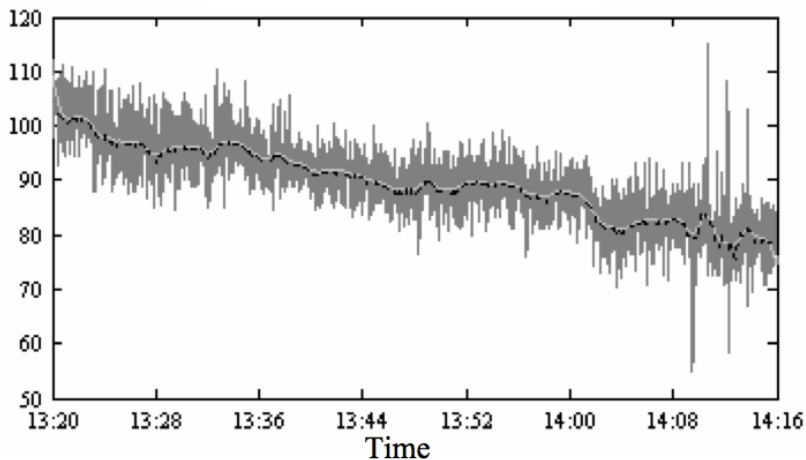
## Features extracted by Honglu

Extract these features from the **entire image**:

- ▶ **A**: number pixels in flame region (area)
- ▶ **B**: average intensity of flame pixels after converting to grayscale (brightness)
- ▶ **U**: standard deviation of grayscale flame pixels (uniformity)
- ▶ **W**: average intensity of background pixels
- ▶  $\mathbf{s}_{1,m}, \mathbf{s}_{2,m} = [ \overline{R}, \overline{G}, \overline{B} ] \mathbf{P} =$  : average colour of entire image projected onto the PCA model
- ▶  $\mathbf{s}_{1,f}, \mathbf{s}_{2,f}$ : as above, except for average flame colour
- ▶ **N<sub>c</sub>**: number of unique colours in the flame region

i.e. generate a feature vector once per second

## Stability of features?

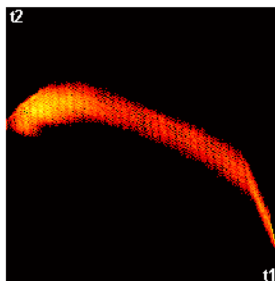


The features are not quite as stable as hoped for. Use smoothing

- ▶ dark-gray: feature from each image (noisy)
- ▶ light-gray: smooth the features after calculating
- ▶ black: smooth in the score space ... described next

# Smoothing

1. Average 60 consecutive images, then calculate features (poor)
2. Calculate features, then use the average (e.g. MA, or EWMA)
3. Calculate 60 consecutive score images, *average in the score space*, then reconstruct image:  $\hat{\mathbf{X}} = \mathbf{t}_1 \mathbf{p}_1^T + \mathbf{t}_2 \mathbf{p}_2^T$  and calculate features on reconstructed image.



Option 2 and 3 give similar performance (previous slide)

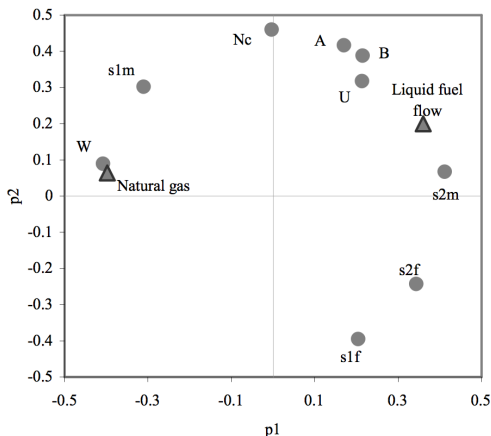
# PCA on the features

## *Waste fuel:*

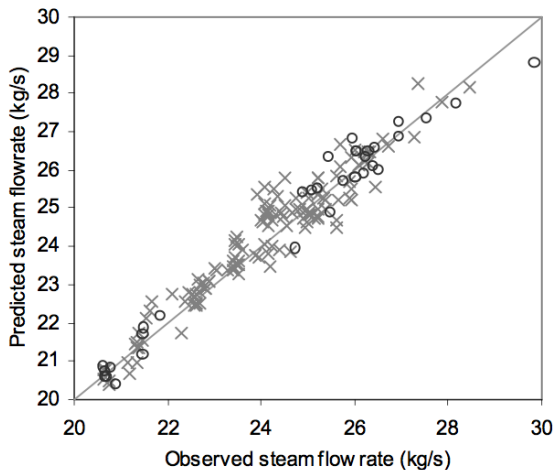
- ▶ bigger flame, brigher, and higher standard deviation (more variable)
- ▶ fewer non-flame pixels
- ▶ higher  $t_2$  values for flame and non-flame regions (shift up-down)

## *Natural gas:*

- ▶ more reflectance off boiler walls (more non-flame pixels)
- ▶ higher  $t_1$  values (shifts left-right)



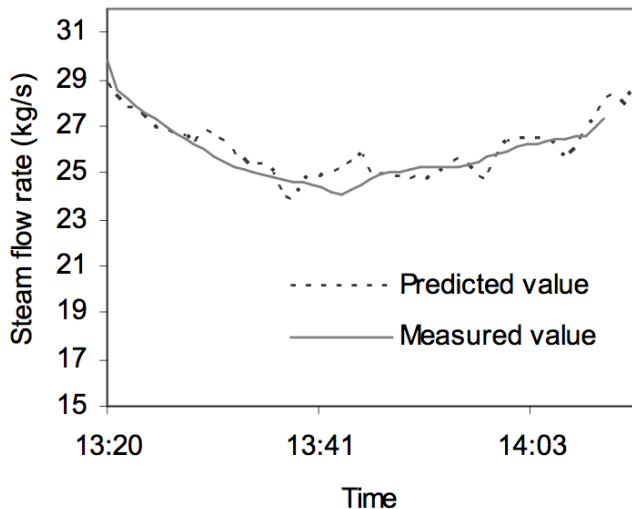
## Prediction results



o = training data (RMSEP=0.4 kg/s)

x = testing data (RMSEE = 0.5 kg/s)

## Prediction results



Definitely would require more in-depth study to prove long-term reliability, but shows good promise.

# What was learned

- ▶ score space has no concept of *spatial* relationships
- ▶ scores only capture *spectral similarities*
- ▶ masking is effective at spectral segmentation (very rapid)
- ▶ hardest part: **good feature extraction**
  - ▶ extract features relevant to goal
  - ▶ features should be as stable as the underlying process

# Seasoning application

We can see a change in product appearance with more seasoning: <sup>2</sup>

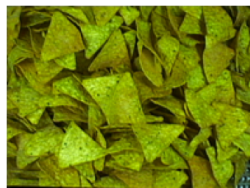


Increased seasoning from left to right

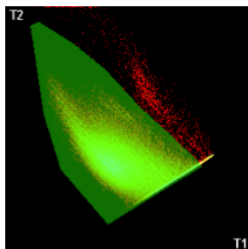
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<sup>2</sup>From [Honglu Yu's PhD work](#) (used with permission)

## Background: segment “background” vs “product” pixels



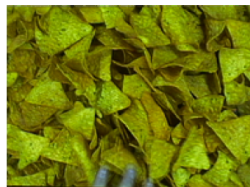
Training image



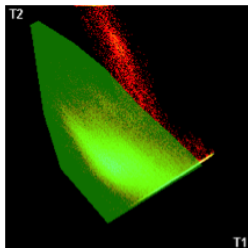
Find suitable mask



Segmented result



Future testing  
image

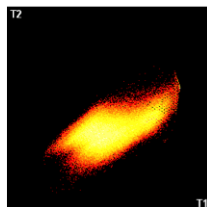
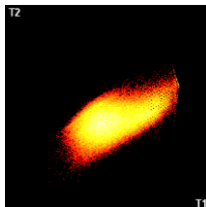
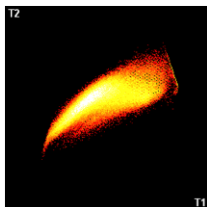
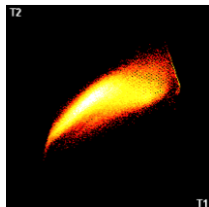


Use the same mask



Segmented result  
on test image

## Score images



**No** seasoning

**No** seasoning

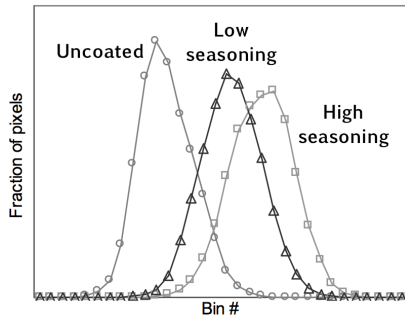
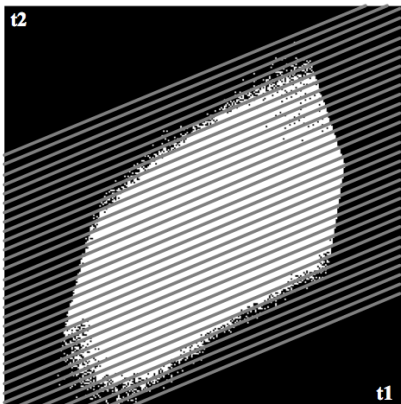
**Low** seasoning

**High** seasoning

- ▶ Image space changes, while score space remains stable
- ▶ Scores change for different seasoning (spectral features)

# Feature extraction in the score space

Create bins in the score space and count pixels



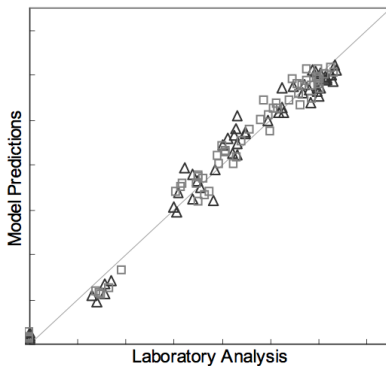
Direction of bins: direction in which seasoning level shifts the score plot <sup>3</sup>

<sup>3</sup>Other binning methods described in [journal publications](#) and thesis

# Model building and predictions

PLS model:

- ▶ **X**-space: cumulative histogram of *fractional* bin counts
  - ▶ Note the **X**-space will be very collinear
- ▶ *y*-variable: seasoning level

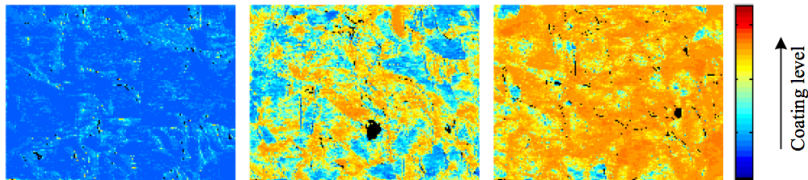


triangles = training data; squares = testing data

## Apply the procedure to new images

Notice that none of the steps taken are a function of the number of pixels.

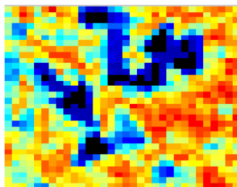
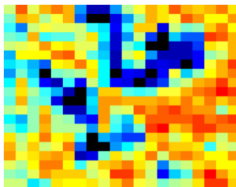
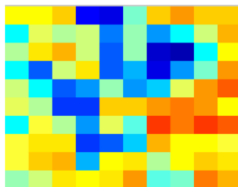
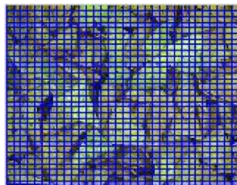
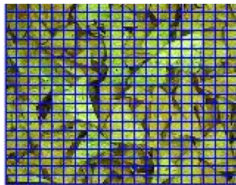
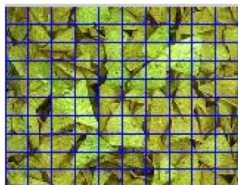
- Can apply the procedure to every pixel in an image



- Used for visual display at the manufacturing line

## Apply the procedure to new images

- ▶ Apply procedure to subset of pixels: “small-window strategy”



- ▶ Calculate seasoning variance: monitoring chart

## Seasoning application: summary

- ▶ Image segmentation again: to remove background
  - ▶ background pixels have different spectral signature
- ▶ Features extracted in the score space this time
  - ▶ score space is stable, while the image space varies
- ▶ Use PLS to regress  $y = \text{seasoning}$  onto  $\mathbf{X} = \text{features}$ 
  - ▶ the features are extremely collinear (MLR would fail)
- ▶ Prediction method is not dependent on the image size
  - ▶ use the “small-window strategy” to predict seasoning variability
- ▶ The seasoning prediction is now used for feedback control at many of the company's manufacturing facilities
- ▶ The seasoning variance can be used in monitoring charts

## Flame application: references

- ▶ Honglu Yu PhD thesis: describes all the details
- ▶ Yu and MacGregor: journal publication
- ▶ Szatvanyi et al. - predict exit temperature from a kiln

Concepts can be applied to any situation where:

- ▶ fast moving and dynamic images, but
- ▶ the process underlying it is inherently slow
  - ▶ Engines, boilers, furnaces, kilns

# Intro to appearance monitoring and control

Manufactured countertops <sup>4</sup>



**Too fine**



**Too high contrast**



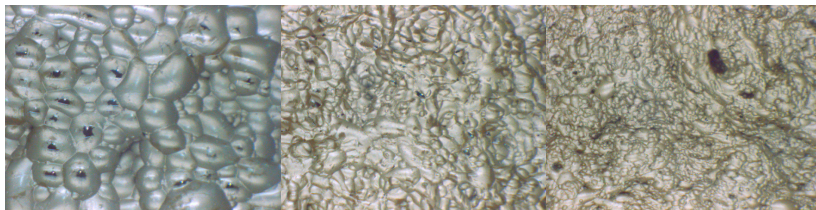
**Too coarse**

---

<sup>4</sup>From [J. Jay Liu's thesis](#) [Ch 8] (used with permission)

# Intro to appearance monitoring and control

## Froth flotation <sup>5</sup>



Poorly-loaded bubbles

Well-loaded bubbles

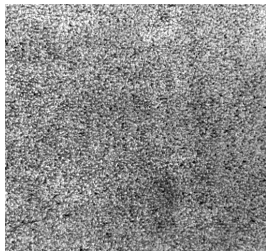
Overloaded bubbles

---

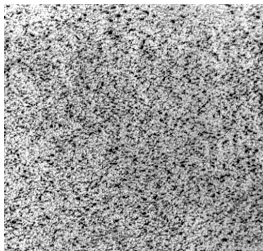
<sup>5</sup>From [June Liu's thesis](#) [Ch 6 and 7] (used with permission)

# Intro to appearance monitoring and control

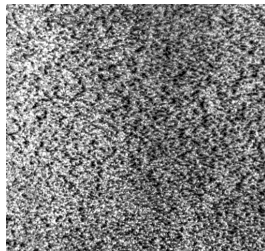
Surface quality of rolled steel sheets <sup>6</sup>



**Good**



**Medium**



**Bad**

---

<sup>6</sup>From **June Liu's thesis** [Ch 3] (used with permission)

# Steel application details

Steel sheets are classified: excellent, good, medium, and bad

- ▶ size and severity of pits
- ▶ number of pits

Excellent and good categories: have only few, shallow pits

## Data available

- ▶ 35 grayscale images from 4 groups (ideally we would like images for testing)
- ▶ black ink is poured on and wiped off: to enhance pits

## Suitable features?

It is the visual appearance (particularly texture) that is important.  
How to quantify this?

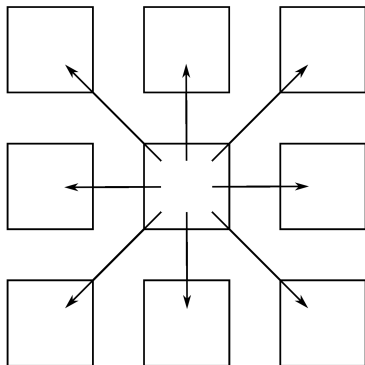
## Extracting texture features: one way

Take the original image and shift it 8 times by 1 or more pixels:

- ▶ Repeat for each colour layer
- ▶ Creates a very correlated cube of data
- ▶ e.g. RGB image: unfolds into  $9 \times 3 = 27$  columns

Some of the scores will capture “directional derivatives”.

More details in [review paper by Prats-Montalbán et al.](#)



## Wavelet texture analysis: another way

Alternative to shifting image is to extract wavelet features.

Decomposes image into 4 parts:

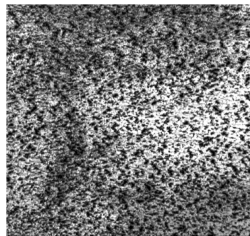
- ▶ a sub-sampled “**approximation**” image (lowpass, then lowpass filter)
- ▶ horizontal details (lowpass, then highpass filter)
- ▶ vertical details (highpass, then lowpass filter)
- ▶ diagonal details (highpass, then highpass filter)

We will combine the 3 “**detail**” images into one for display

$$\mathbf{X} = \mathbf{approximation} + \mathbf{detail}$$

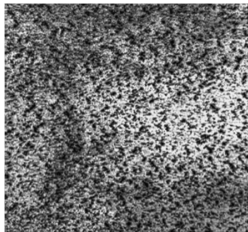
**Caution:** I'm an extreme novice wrt wavelets ... use this section with caution.

## Wavelet texture breakdown on bad steel image



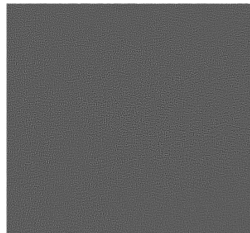
Original

=



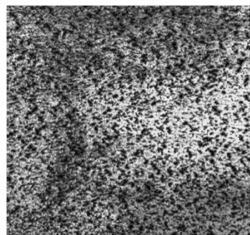
A1

+



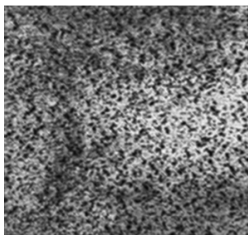
D1

## Wavelet texture breakdown on bad steel image



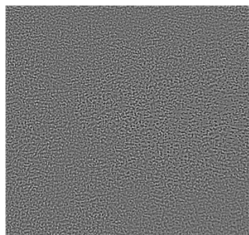
A1

=



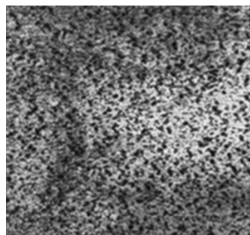
A2

+



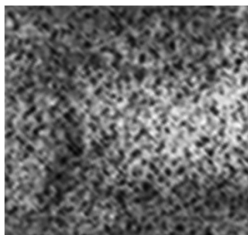
D2

## Wavelet texture breakdown on bad steel image



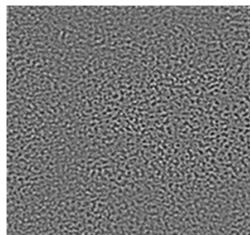
A2

=



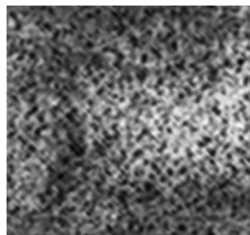
A3

+



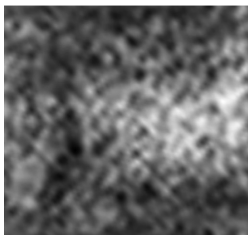
D3

## Wavelet texture breakdown on bad steel image



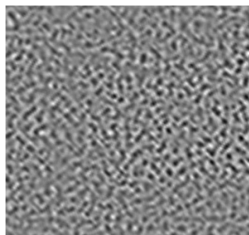
A3

=



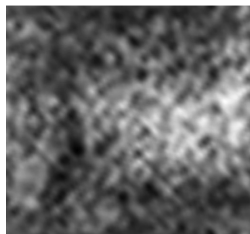
A4

+



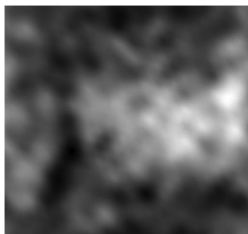
D4

## Wavelet texture breakdown on bad steel image



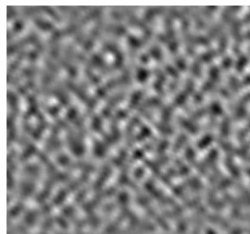
A4

=



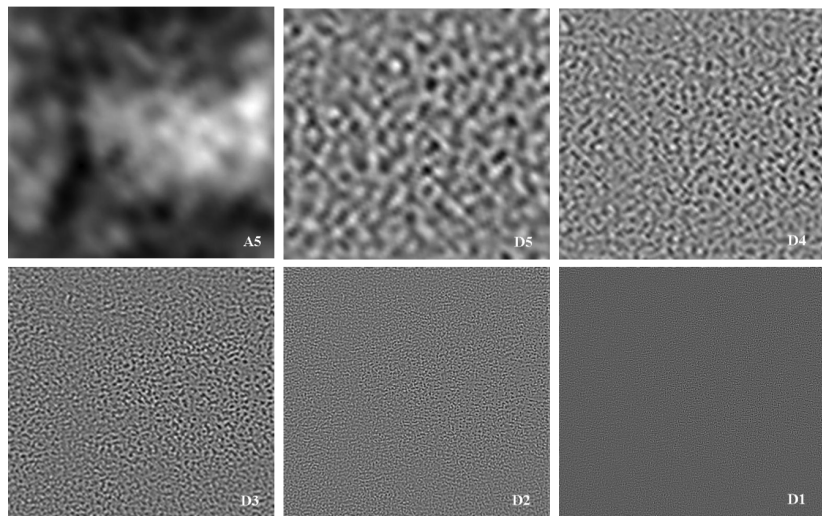
A5

+



D5

## Wavelet texture summary



A complete summary of the original image at different frequencies.

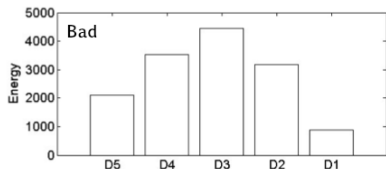
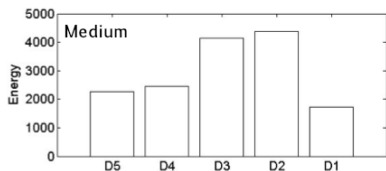
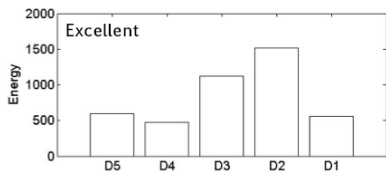
# Feature extraction

- ▶ D1, D2, ... D5: capture lower and lower frequencies
- ▶ D1: captures really fine, high frequency details
- ▶ Larger pits are captured in higher  $D_i$  images

Potential features to extract from  $D_i$  images:

- ▶ “energy” =  $\sqrt{\text{sum of squares of pixel values}}$  = variability
- ▶ “entropy” = sum of absolute values ( $0 \implies$  a solid colour)
- ▶ “maximum” and “minimum”

# Examples of features extracted



As expected, bad surfaces have more energy in the higher details (larger pits)

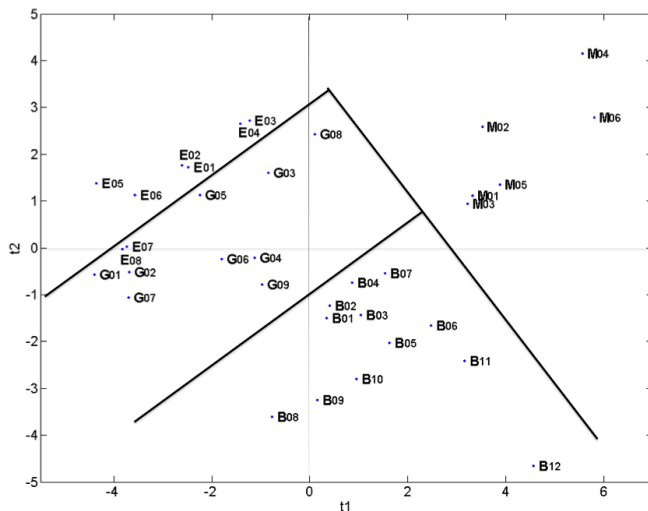
June Liu extracted 15 *energy* features:  $5 \times 3$  details:

1. horizontal
2. vertical
3. diagonal

The sum of the energy in all 3 directions is shown on the left.

Calculations in MATLAB;  
used Coiflet wavelet.

# Separating grades of steel



- ▶ PCA on the 15 features show good, natural separation into the 4 categories: **E**xcellent, **M**edium, **G**ood and **B**ad

- ▶ Separating lines are added manually

## Lighting, lighting, lighting

- ▶ surface pits on steel: how to emphasize pits?
- ▶ snack food: performance degrades dramatically if ambient light changes

# Practical issues

Drift in ambient lighting (day/night)

- ▶ Recalibrate with a neutral background periodically
  - ▶ background belt (e.g. seasoning application)
  - ▶ colour cards of known spectral intensity

Correct for drift. More details in [Zheng Liu's Masters thesis](#).

Similar issue: different cameras give different pixels values on same image scene

# Summary

Feature extraction step is critical:

- ▶ Assumption is that image which you extract features from is entirely related to  $y$
- ▶ That is only an approximation
  - ▶ seasoning in image not all the same
  - ▶ surface appearance not all the same
- ▶ The stronger this assumption is met, the better the predictions
- ▶ Extract features from the space that varies with the  $y$ -variable
  - ▶ flame application:  $y$  was stable in one frame to the next, feature space should also be